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Developing modelling competencies in pre-service mathematics teachers through reflective thinking skills

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Background: Ghana's commitment to quality education has been reflected in its goal of providing equal access to high-quality education, leading to the reforms of the New Common Core Mathematics Curriculum for Basic Schools. The study explored the integration of mathematical modelling using reflective thinking skills, which are not currently core competencies in Ghanaian basic school curriculum.

Aim: The study examined the modelling proficiency of pre-service mathematics teachers by assessing their reflective thinking abilities in a modelling laboratory context.

Setting: The study focussed on pre-service mathematics teachers from two education colleges in Ghana.

Methods: Using purposive sampling to select participants, a quasi-experimental design with pre- and post-test interventions was employed. Data were analysed through content and inferential analysis, supplemented by interviews.

Findings: The findings indicated that the comparison group lacked prior knowledge of modelling problems and struggled with comprehension tasks. In contrast, the experimental group successfully translated real-world problems into mathematical models.

Conclusion: Providing pre-service mathematics teachers access to a modelling laboratory and modelling-eliciting activities was essential for developing future modellers. This approach would enhance their effectiveness in teaching foundational mathematics in Ghanaian education.

Contribution: This study advocated for re-orienting the mathematics curriculum at both Basic schools and Colleges of Education in Ghana to include mathematical modelling and reflective thinking skills as core components.

Keywords: developing modelling; modelling competencies; reflective thinking skills; modelling laboratory; pre-service mathematics teachers.

Introduction

This article describes a study that explored the impact of a modelling laboratory on the reflective thinking and modelling proficiency of pre-service mathematics teachers in Ghana. It begins with a background on the importance of mathematical modelling in education, followed by research questions and the hypothesis, focussing on assessing how reflective thinking skills can enhance modelling abilities. The article then details the research methodology, including a quasi-experimental design with pre- and post-tests. It presents the key findings, highlighting differences in performance between the experimental and comparison groups. Finally, the article discusses the implications of these findings for mathematics education in Ghana and offers recommendations for integrating modelling into the curriculum.

In today's mathematics education, pre-service mathematics teachers play a crucial role in their ability to engage in mathematical modelling and develop reflective cognitive skills. Mathematical modelling serves as a creative strategy and a fundamental concept that enhances mathematical knowledge, giving the learning process a clear purpose (Salha & Qatanani 2021). It involves the process of translating a real-world situation into a mathematical model. However, the mathematics laboratory is more than just a space. It is a structured set of activities designed to help students construct meaningful mathematical concepts through hands-on learning, observation and communication with peers and experts. According to Maschietto and Trough (2010), the mathematics laboratory is integral to teacher education's perspective and practice.

The concept of a mathematics laboratory did not originate from pedagogical research but from the reflections of mathematicians concerning the use of artefacts. Global studies on the mathematical modelling process emphasise the importance of reflective thinking (Thahir et al. 2019; Yasin et al. 2020). However, there is a notable gap in research focussed on pre-service mathematics teachers and the tasks leading to creating a modelling laboratory to support the development of reflective thought processes and modelling proficiency. Establishing a learning environment that fosters reflective thinking and understanding the foundations of this skill are complex tasks (Davydov & Rubtsov 2018). Reflection is a central element in teacher education and the professional development of aspiring pre-service mathematics teachers (Agustan, Junniati & Siswono 2017; Amidu, 2012; Lim 2011).

Strong content-oriented knowledge underpins pre-service mathematics teachers' proficiency in modelling (Gocheva-Ilieva et al. 2018; Govender 2018). Besides broadening mathematical understanding and providing direction in the learning process, mathematical modelling is a tool for creativity (Salha & Qatanani 2021). According to Rellensmann, Schukajlow and Leopold (2017), pre-service mathematics teachers' proficiency in solving modelling tasks increases as they become skilled in creating and using diagrams to represent these tasks.

Mathematical modelling, which involves applying mathematics to real-world problems, benefits significantly from developing reflective thinking skills. These skills can be purposefully cultivated through experimental approaches, such as designing and facilitating problem-based learning, simulations and collaborative learning activities. These approaches would help pre-service mathematics teachers develop the ability to understand and tackle complex modelling tasks (Davydov & Rubtsov 2018; Yasa & Karatas 2018).

Analysis can be considered an experimental approach when it is used to explore how people think and solve mathematical problems. It helps uncover the relationships or principles needed to solve a problem and understand the factors involved (Davydov & Rubtsov 2018:303). Furthermore, analysis helps pre-service mathematics teachers practise and improve their skills in theoretical and practical modelling activities (Borromeo-Ferri 2018), building reflective thinking skills to solve complex mathematical problems.

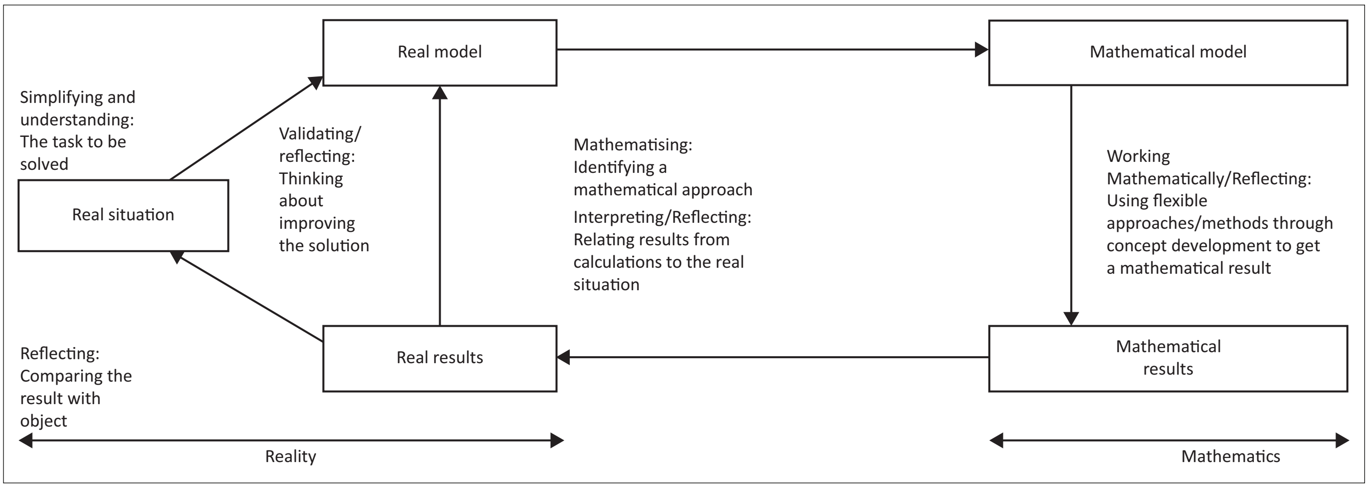
Modelling is not listed as a core competency in the newly implemented 2019 curriculum or explicitly mentioned in the mathematics curriculum. However, Ghanaian education colleges should include modelling laboratories because, as Durandt (2021) suggests, they foster a positive attitude towards learning. Pre-service mathematics teachers need mentoring, which involves applying mathematics to simple and complex real-world problems, bridging the gap between everyday and academic discourse (Durandt 2021).

To achieve this, the pedagogical modelling cycle proposed by Mhakure and Jakobsen (2021) should be modified to incorporate reflection and innovative approaches (Lu & Kaiser 2022). An accurate interpretation obtained through reflection would allow the solution to be validated, and the outcome can be re-examined using the model. Pre-service teachers must repeat the modelling cycle if the procedure does not accurately estimate the findings. They should use models to solve problems and carry out the necessary steps of the modelling process (Rellensmann, Schukajlow & Leopold 2020). However, pre-service mathematics teachers may find modelling challenging because of a lack of experience or limited professional knowledge (Breiner et al. 2012; Corlu & Capraro 2014).

Asante and Mereku (2012) assert that pre-service mathematics teachers do not have sufficient time to practice teaching at the foundational level of Ghanaian education. Moreover, pedagogical approaches are not given adequate consideration. This lack of practical teaching experience and emphasis on pedagogy extends to mathematical modelling, where teachers may struggle to apply theoretical concepts in real-world contexts. Without adequate practice in modelling, these teachers may find it challenging to develop the reflective thinking skills necessary to integrate modelling into their teaching, limiting their ability to prepare students for complex problem-solving tasks.

In the 2011 Trends in International Mathematics and Science Study (TIMSS) assessment, Ghanaian students consistently scored below international benchmarks. Therefore, another assessment, the Early Grade Mathematics Assessment (EGMA), was conducted in 2015 to diagnose the problems of early-grade students concerning basic mathematical skills and competencies (Armah & Mereku 2018). The findings revealed that only 25% of students could accurately answer questions in the conceptual knowledge subtasks, compared to 46%–72% in the procedural knowledge subtasks. Furthermore, the Basic Education Certificate Examination (BECE) results showed that students' performance was low (Akyeampong 2017; Asante & Mereku 2012).

Therefore, pre-service teachers were recommended to take practical courses in mathematics pedagogy that offer ample opportunities to practice teaching at the basic level of Ghanaian education. Mathematical modelling is considered a challenging procedure, particularly when understanding, simplifying, synthesising and realising the problem within given constraints (Govender 2020; Govender & Machingura 2023). The study sought to investigate how pre-service mathematics teachers could use their reflective thinking skills to solve problems related to mathematical modelling. In addition, the establishment of a modelling laboratory at



Source: Mhakure, D. & Jakobsen, A., 2021, 'Using the modelling activity diagram framework to characterise students' activities: A case for geometrical constructions', in F.K.S. Leung, G.A. Stillman, G. Kaiser & K.L. Wong (eds.), *Mathematical modelling education in East and West. International perspectives on the teaching and learning of mathematical modelling*, p. 415

FIGURE 1: Modified pedagogical modelling cycle.

education colleges was proposed, along with a restructuring of the mathematics curriculum to include modelling as one of the core competencies.

Research questions and hypothesis

This study explored the development and application of reflective thinking skills in pre-service mathematics teachers, particularly within mathematical modelling. The following questions guided the research:

1. *How do pre-service mathematics teachers use their reflective thinking skills to solve real-life problems in a mathematical modelling context?*
2. *What is a modelling laboratory's relevance in developing pre-service mathematics teachers' reflective thinking skills when solving modelling problems?*

The following hypothesis ties together the research questions by testing whether a modelling laboratory significantly enhances reflective thinking skills in pre-service mathematics teachers. It checks explicitly if there is a measurable difference between those with and without access to the laboratory:

> H_0: The reflective thinking skills of pre-service mathematics teachers in the experimental and comparison groups employing the modelling process technique do not differ significantly.

Research methods and design

A pre-test and post-test quasi-experimental design was used in this study. According to Johnson and Christensen (2012), quasi-experimental research provides the most robust evidence for cause-and-effect correlations from manipulating and controlling irrelevant variables. The quasi-experimental research methodology was employed in this study, involving two Colleges of Education: one providing the experimental group and the other the comparison group to evaluate the intervention's impact on B.Ed. students.

Mathematics pre-service teachers were sampled based on their willingness instead of random selection (Johnson & Christensen 2012). However, a quasi-experimental design may offer weak evidence of a causal relationship between variables because there is no random assignment to groups or manipulation of the independent variable (May 2017). Nevertheless, this design is essential in educational research because many research questions in education do not lend themselves to experiments (Creswell & Creswell 2018).

Using a purposive sample, 35 pre-service mathematics teachers were selected from a Ghanaian College of Education for the comparison group and 38 pre-service mathematics teachers were selected from another Ghanaian College of Education for the experimental group. Using semi-structured interviews and tests as data-collecting instruments, the participants were divided into seven groups of five using the random number table. Mhakure and Jakobsen's (2021) theoretical and pedagogical modelling frameworks were modified to address the research questions and the hypothesis (see Figure 1 and Table 1). The most important feature of the theoretical framework was the skill of reflective thinking, which is applied in all the stages of the modelling cycle. Moreover, applying innovative, flexible approaches to develop the mathematical model, followed by reflection and interpretation of the results, validated and refined the model, ensuring it reflected the real-world scenario (see Figure 1).

A pedagogical modelling activity framework also guided the process that pre-service mathematics teachers used to obtain accurate estimates of their learning outcomes. This was achieved by applying the pedagogical mathematical activity (PMAD) framework outlined in Table 1, categorising the activities involved in the modelling process.

These activities were successful based on an intervention modelling activity lesson plan detailing the critical aspects regarding mathematical activity and modelling activity (see Table 2). The intervention took 8 weeks of face-to-face instruction for B.Ed. Mathematics Education pre-service mathematics teachers for the 2021–2022 academic year at the Colleges of Education in Ghana. During the intervention, the

TABLE 1: Pedagogical mathematical activity framework.

Categories of PMAD	Description
Reading	Understanding and unpacking the information
Modelling	Changing the task's context from the real world to a mathematical model
Estimating	Making meaning of the problem's quantitative estimations in the context
Calculating	Calculating the missing data on the drawn diagram using simple mathematical ideas
Reflecting	Identifying mathematics concepts, facts, formulas and theorists relevant to the task solution
Validating	Interpreting, verifying and validating the results, calculations and models in a real-world setting
Writing	Providing a succinct explanation of a report's findings, how they relate to the original task, and the methods that led to the task's solution

Source: Mhakure, D. & Jakobsen, A., 2021, 'Using the modelling activity diagram framework to characterise students' activities: A case for geometrical constructions', in F.K.S. Leung, G.A. Stillman, G. Kaiser & K.L. Wong (eds.), *Mathematical modelling education in East and West. International perspectives on the teaching and learning of mathematical modelling*, p. 416
PMAD, pedagogical mathematical activity.

TABLE 2: Sample intervention modelling lesson activity.

Task	Reflective thinking	Modelling	Mathematical solutions	Final analysis
1. An aeroplane flying at a height of 650 m has an angle of elevation of 50° measured from the runway. What horizontal distance must the aeroplane cover before it reaches the runway, correct to the nearest whole number? 2. A cable car takes tourists to the top of a mountain. The cable is 2.3 km long and makes an angle of 43° with the ground. What is the height of the mountain, to the nearest metre?	**Pre-service mathematics teachers will be able to:** Review previous concepts, subject matter and theories in reading and understanding the task. Recall key concepts related to the task by identifying facts, formulas and theories that are related to the task	**Pre-service mathematics teachers' activities include:** To demonstrate knowledge in representing the real situation task from a real model to a mathematical model through mathematising, simplifying and understanding the task. To develop concepts using the modelling process cycle	Pre-service mathematics teachers apply flexible methods and approaches or theories in solving tasks to get the right answers	Pre-service mathematics teachers apply reflection (looking back and learn, unlearn and relearn) by interpreting, verifying and validating the results. Provide a succinct explanation of the findings and how they solved the problem solution through PowerPoint presentations

comparison group solved the tasks using the conventional approach through reflective thinking, recalling concepts and theories to apply to the real-world scenario. However, the experimental group solved the task using theoretical and pedagogical activity frameworks. Furthermore, the experimental group worked in a well-planned and conducive modelling laboratory. After the problem-solving, the groups did PowerPoint presentations of their findings on a whiteboard.

Table 2 presents a sample of what and how pre-service mathematics teachers conducted the intervention through modelling.

Table 1 and Table 2 show the conceptual framework of the experimental group work, outlining reflective thinking and mathematical modelling. The experimental group members applied the concept of reflective thinking to review previous concepts, subject matter and theories to facilitate their comprehension of the task. Thereafter, they demonstrated knowledge by representing the real-life problem as a mathematical model through mathematisation and simplification. Their next task was to apply flexible methods or suitable theories to solving the mathematical problem embodied in the model. Finally, they applied reflection by looking back to learn, unlearn and relearn through interpreting, verifying, validating and communicating how they turned the real-life problem into a mathematical model, indicating the methods and theories that led to the result.

Ethical considerations

The Humanities and Social Sciences Research Ethics Committee for Ethical Research (HSSREC) at the University of the Western Cape was consulted, and an ethics clearance certificate was issued with the HSSREC reference number HS22/6/54 before the researcher began collecting data. Moreover, permission approval letters were obtained from the Director General of the Ghana Tertiary Education Commission, principals from the sampled Colleges of Education, the Heads of the Department of Mathematics/ICT and the tutor trained for the intervention. Students signed consent forms indicating their willingness to be part of the study. They were assured of confidentiality and informed that it was for research purposes only.

Results

Quantitative and content analyses were conducted on the pre-test scores of the comparison and the experimental groups, and the quantitative results and qualitative findings are presented and discussed in detail.

Table 3 presents the quantitative results of Levene's test for equality of variance for the pre-test scores and the t-test for equality of means between the experimental and comparison groups. These statistical tests assessed whether there were significant differences in variance and mean scores between the two groups before the intervention.

Table 3 indicates that with t (0.246), df (12) and a p-value of 0.81, greater than the alpha level of 0.05, there was no statistically significant difference in the pre-test scores of the comparison and experimental groups. Furthermore, Levene's test for equality of variance, with F (0.013) and a p-value of 0.91, confirmed no significant differences in variance of the pre-test results for the two groups.

Pre-test answers were subjected to content analysis to determine the pre-service mathematics teachers' competency levels in applying reflective thinking skills to mathematical tasks. The comparison group was classified as groups A, B, and so on, while the experimental group was classified as

TABLE 3: Levene's test of equality of variance of pre-test scores.

Groups	Levene's test of equality of variance		*T*-test for equality of means		
	F	Sig.	*t*	*df*	*p*
Experimental and comparison groups	0.01	0.91	0.25	12	0.81

df, degrees of freedom.

groups 1, 2, and so on. This approach was based on the understanding that, through extensive reflection, the pre-service teachers would start with simple real-world problems, applying foundational concepts. As their knowledge deepened, they would move on to complex tasks, applying increasingly advanced mathematical concepts, including those used in trigonometry (Tan & Ang 2012).

The first research question was:

> *How do pre-service mathematics teachers use their reflective thinking skills to solve real-life problems in a mathematical modelling context?*

To address this question, the study focussed on observing their problem-solving approaches and subsequent reflections. The research explored whether reflective thinking enabled these pre-service teachers to connect theoretical knowledge with practical application, adapt their strategies and improve their modelling processes over time. The findings, detailed in the subsequent sections, reveal how reflective thinking facilitated their ability to break down complex problems, apply mathematical concepts effectively, and refine their approaches to achieve accurate and relevant solutions in real-world scenarios.

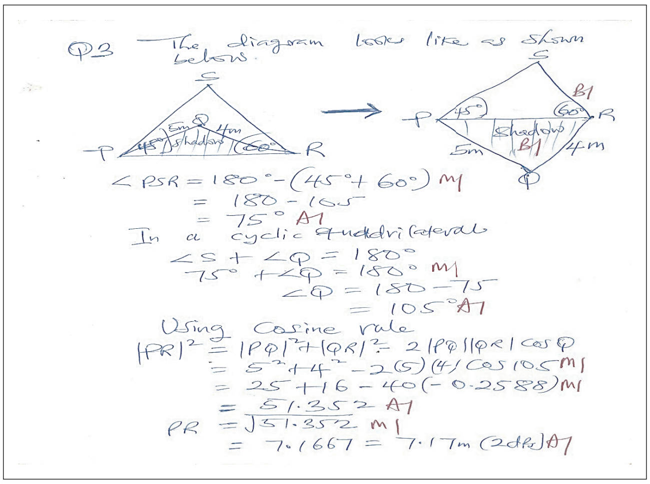
The findings revealed that every group refrained from attempting Task 3, except for those in Comparison Group G, who had an idea and could unearth pertinent data to solve every task based on assumptions from trigonometry concepts. However, while employing relevant mathematical approaches to tackle the challenges, this group did not effectively utilise reflective thinking skills. A sample of Group G's solutions in Task 3 is illustrated in Figure 2. The scoring was carried out using the following criteria: B for Basic Concept or Best Accuracy, M for Method, A for Answer and NJ for Not Judicious (indicating that the answer was correct, but the basic concept or method applied was incorrect), as shown in the sample solutions in Figure 2.

Task 3's instructions were as follows (see Appendix 1):

3. A spotlight for a theatre production illuminates a triangular area on stage. Actors are to stand at the corners of the illuminated area at P, Q and R. The actors at P and R have to stand 5m and 4m away from the actors at Q, respectively. The angle of elevation of S from P is 45°, and the angle of S from R is 60°. If the spotlight is placed at a point vertically above PR.

Draw a diagram to illustrate the given information.

Determine how the actors at P and R must stand from each other.



Source: Pre-service mathematics teachers sample response

FIGURE 2: Comparison Group G's solution for Task 3.

The actor at P enters on stage by sliding down a wire from S to P. How long is the wire that the actor slides along?

Figure 2 presents Group C's solution to Task 3. The diagram demonstrates Group C's calculations for the actors' placement based on the given distances and angles of elevation. Additionally, it shows their method for determining the distance between the actors at P and R, as well as the length of the wire along which the actor at P slides down from point S. The diagram effectively captures the geometric relationships and trigonometric calculations used by Group C to solve the task, providing a visual representation of the problem and their approach to the solution.

Group G of pre-service mathematics teachers in the comparison group was able to conceptualise the real scenario simply and construct the trigonometric model, as shown in Figure 2. After calculating the angle at Q using the idea of cyclic quadrilateral production as a result of the spotlight, they employed trigonometry and discovered that the angle was 105°. They built a trigonometric diagram to mathematically represent the real-life situation using notation and measurements like 4 m and 5 m to illuminate a shadow from the theatre production. With this model's aid, students could access and deconstruct the necessary mathematical knowledge to generate a solution.

They could determine the angle at Q by adding the opposite angles in a cyclic quadrilateral using the keyword 'spotlight' that was generated to the cycle. With the help of this result, they could use the Cosine rule to determine that the PR length was 7.17 m. This finding demonstrated that pre-service mathematics teachers (1 out of 7 groups) could evaluate, check and reflect on the discovered answer to the extent that they could examine their trigonometric diagram and had the correct outcome from the spotlight on the theatre production.

Task 1 required the students to determine the value of the unknown variable in a triangle correct to one decimal place.

However, because of their inability to apply the assumptions of the Sine and Cosine rules, Comparison Group C divided the given triangle in half to produce a right-angled triangle (see Figure 3 and Figure 4).

Instead of using the Sine and Cosine rule, the members of Group C applied the Pythagoras theorem, which produced the correct answer and is also considered logically and mathematically accepted because the vertical line meets the base at 90°, creating a right-angled triangle. However, the group's approach might have prevented them from completing the other tasks because Tasks 2 and 3 were beyond their comprehension because they could not apply the Sine and Cosine rules.

Because of a lack of concept application, two comparison groups could not do Tasks 2 and 3. Although they applied previously taught trigonometric functions, they failed to reflect on and integrate their prior knowledge and experience with these concepts. These groups correctly drew the diagram for Task 3 but did not find the solution because they could not locate the correct number of angles. However, one particular group correctly drew the diagram for Task 3 using a novel approach but could not find the solution. Most groups approached the different tasks in conventional ways and could not apply the mathematical modelling processes.

The experimental groups could do Task 1 by applying assumptions and mathematical skills. However, experimental groups 2, 4 and 6 could not do tasks 2 and 3, while experimental groups 3 and 5 did not even attempt them. Experimental Group 7 formulated prepositions, facilitating their completion of Task 1. However, as explained earlier in the intervention stage, they did not think critically or creatively when making the diagram for Task 3. They used the wrong concept in the wrong diagram, which led to a correct answer, which was not judicious. Even though this approach helped them answer other tasks correctly, they were only given marks for the method they used, not for the accuracy of the answer, as shown in Figure 5.

Experimental Group 7 created the Task 3 diagram using a novel drawing technique but could not show clearly how the spotlight casts a shadow to form the quadrilateral. A mark for the method was given, but they were denied the answer mark. Although the answer was correct, it was not judicious because concepts were missing from the diagram. They understood the new concepts, correctly applied the Cosine rule, and they justified their answer. However, they eventually had to acknowledge that the missing basic concept (B) and method (M) marks indicated that they had not solved the problem. In addition, a not judicious (NJ) mark was subtracted from the answer (A) mark, suggesting that while the answer was technically correct, the approach was inappropriate.

An analysis of the covariance (ANCOVA) test was performed to examine the differences in post-test scores between the groups while controlling for any initial differences in the pre-test scores. This test would quantitatively address the hypothesis that the reflective thinking skills of pre-service mathematics teachers in the experimental and comparison groups employing the modelling process technique do not differ significantly.

Table 4 presents the ANCOVA results for both the experimental and comparison groups. The table provides a detailed view of the observed differences' statistical significance and effect size.

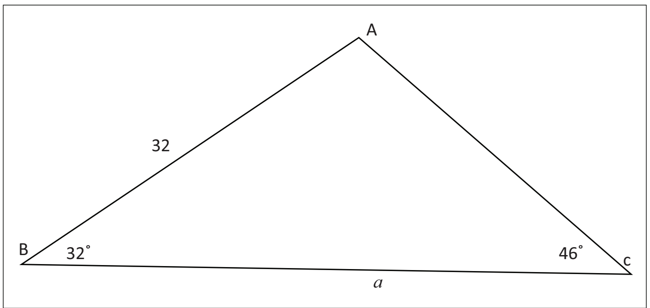
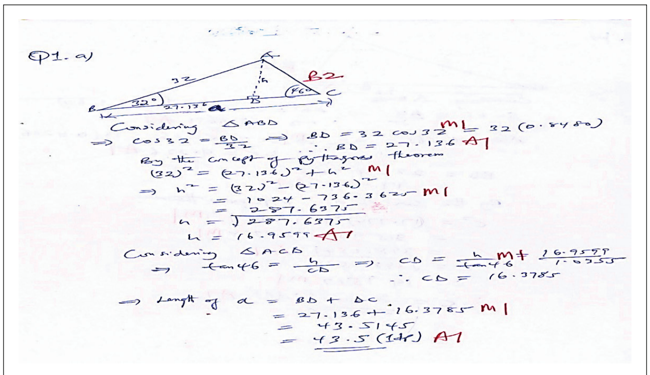
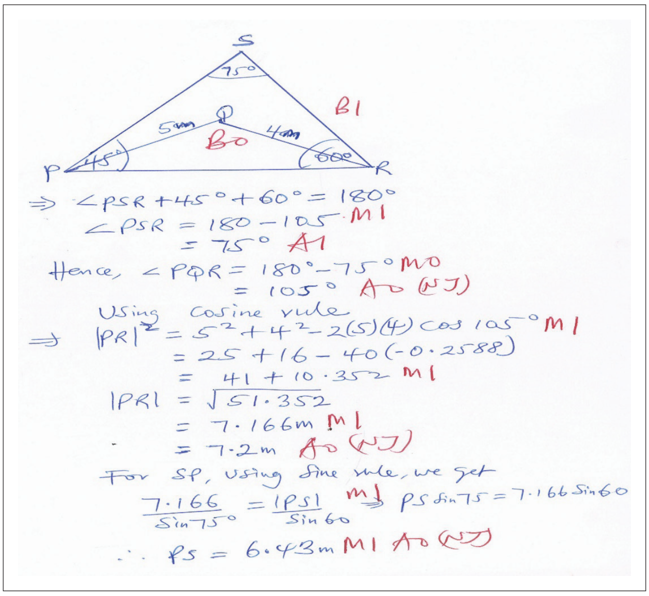


FIGURE 3: Comparison Group C's Task 1.



Source: Pre-service mathematics teachers sample response

FIGURE 4: Comparison Group C's solution to Task 1.



Source: Pre-service mathematics teachers sample response

FIGURE 5: Experimental Group 7's solution for Task 3.

TABLE 4: The analysis of the covariance test results for experimental and comparison groups.

Dependent variable: Experimental and comparison group						
Source	**Type III sum of squares**	***df***	**Mean square**	***F***	***p***	**Partial eta squared**
Corrected model	157.79	1	157.79	6.00	0.03	0.33
Intercept	845.26	1	845.26	32.13	0.00	0.73
Group	157.79	1	157.79	6.00	0.03	0.33
Error	315.71	12	26.31	-	-	-
Total	3417.00	14	-	-	-	-
Corrected total	**473.50**	**13**	-	-	-	-

df, degrees of freedom.

Table 4 shows a statistically significant difference between the experimental and the comparison groups in how they invoked their reflective thinking abilities when solving problems using the modelling process because the p-value of 0.031 was lower than the alpha value of 0.05. While both groups could think reflectively when responding to modelling tasks, the partial eta squared of 0.33 indicated a significant effect size on the modelling process on the reflective thinking ability of pre-service teachers in the experimental group.

This result implied that the null hypothesis was not accepted and that there was no difference between the experimental and comparison groups in their reflective thinking when solving problems through mathematical modelling. However, juxtaposing the post-test results of the groups indicated that the experimental group used reflective thinking, producing accurate mathematical models in Tasks 1 and 2.

Figure 6 presents a side-by-side boxplot illustrating the impact of the 8-week intervention on the post-test scores of both the comparison and experimental groups. This visual representation allows for a clear comparison of the score distributions between the two groups, highlighting the extent of the intervention's effect.

Group 6 completed all tasks successfully by utilising relevant modelling skills. However, three other groups chose not to undertake Task 3 because of a lack of modelling proof. In other words, they did not have sufficient understanding, reasoning or evidence to attempt to solve the task. They were unsure how to justify their methods or solutions using the appropriate mathematical concepts and reasoning.

Both groups applied modelling techniques, but the experimental group demonstrated a deeper understanding and more accurate application of the concepts, leading to better results. The experimental group saw the distinction between solving word problems and modelling. They understood that solving word problems involves applying a straightforward, often pre-taught formula to arrive at a solution. In contrast, modelling requires creating a mathematical representation of a real-world scenario, interpreting the situation, making assumptions and refining the model to find a solution. Modelling requires critical thinking, creativity and connecting mathematical concepts with real-world situations. It involves creatively combining and adapting multiple mathematical tools to develop a new, composite solution that accurately represents a real-life problem.

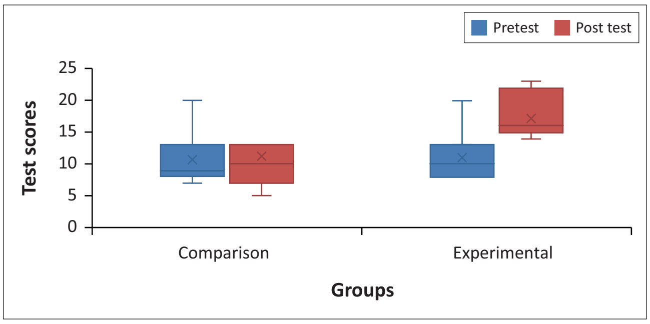


FIGURE 6: A side-by-side boxplot of the groups' test scores.

The experimental group extensively utilised reflective thinking, progressing from simple to complex approaches enhancing their proficiency in modelling. They successfully applied flexible methods alongside the modelling cycle and recognised that reflective thinking is integral to every aspect of the process. This realisation underscored the importance of establishing modelling laboratories in Colleges of Education in Ghana, addressing the second research question:

> *What is a modelling laboratory's relevance in developing pre-service mathematics teachers' reflective thinking skills when solving modelling problems?*

A modelling laboratory would enhance reflective thinking skills by providing a controlled environment where pre-service teachers can engage in hands-on, practical activities that promote reflection, critical thinking and applying theoretical knowledge to real-world problems. This structured setting would help teachers refine their problem-solving approaches, leading to an understanding and mastery of mathematical modelling, which is essential for effective teaching in the classroom.

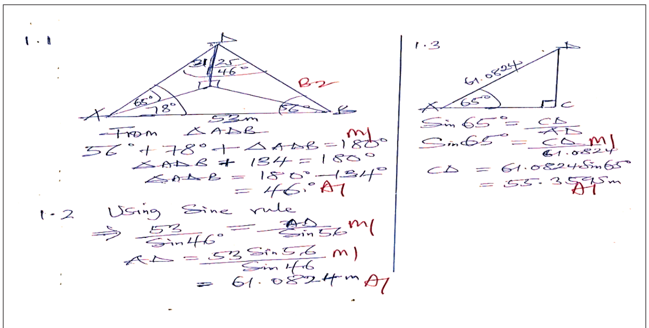
The first experimental group comprehended and analysed the essential details of all tasks. They organised their thoughts and transformed the real-world problem into mathematical models by creating appropriate diagrams. Although some concepts in Subtask 1.3 led to erroneous substitutions and incorrect answers, the students accurately derived the necessary relationships by methodically applying relevant ideas and measurements in their calculations. Additionally, the group utilised a trigonometric model to enhance their understanding of the modelling process. This approach resulted in accurate solutions for both Tasks 1 and 2. Group 1 particularly valued the modelling process, which helped them isolate essential data, apply facts and formulas, and understand the underlying concepts. For instance, in Task 3, they successfully used the Pythagorean theorem by identifying right-angled triangles within the figure.

Experimental Group 1 derived a model to solve the problem of the distance between two cliffs using trigonometric relationships. This is indicated here as:

The students first established the equation $DE = \frac{EF}{Sin\alpha} \Rightarrow EF = DESin\alpha$. They deduced that $BE = BF + EF$ but $BF = y$ and $EF = DESin\alpha$ *as* Equation 1. They then revealed that $tan\theta = \frac{BE}{BC}$ but $BC = x \Rightarrow tan\theta = \frac{BE}{x} \Rightarrow BE = x\,tan\theta$ as Equation 2 equating their two modelling equations, which resulted in the following model: $y + DESin\alpha = x\,tan\,\theta \Rightarrow DESin\alpha = x\,tan\theta - y$. Thus, they formulated the required model relation as $DE = \frac{x\,tan\theta - y}{Sin\,\alpha}$. Substituting the correct parameters or measures, the distance between the cliffs was found to be 750 m.

Members of Group 2 approached Task 1 by leveraging reflective thinking to bridge the gap between a real-world scenario and a mathematical model. They successfully calculated the necessary measures and accurately represented them through mathematisation, creating an appropriate model for Task 1. Their creativity enabled them to identify right-angled triangles within the diagram, which they used to solve subtask 1.3 and obtain the correct answer. By reflecting on theorems and trigonometric models, as illustrated in Figure 5 (see Appendix 2), they effectively interpreted, verified and validated their results.

Figure 7 illustrates the step-by-step process followed by Group 2 in solving Task 1. The diagram on the left (1.1) shows the application of angle properties within triangles to determine an unknown angle in the triangle. In step 1.2, the Sine rule was employed to calculate the length of side AD in the same triangle. Diagram 1.3 demonstrates how Group 2 used trigonometric principles, precisely the Sine function, to find the size of the side CD in the right-angled triangle. The remarks of the variables in red indicate critical points in their solution, such as the correct application of theorems and the steps leading to the accurate calculation of the required measures.



Source: Pre-service mathematics teachers sample response

FIGURE 7: Experimental Group 2 sample post-test solution.

For Task 2, Group 2 created an appropriate diagram using the same conceptualisation techniques and reflective thinking. The group was able to apply trigonometric ratios in determining the length of AD regarding y and α as seen as $= \frac{y}{tan\,\alpha}$. Similarly, for the length of BD regarding y and Θ, they obtained $BD = \frac{y}{tan\theta}$. For Task 3, group members successfully answered Subtask 3.1 and deduced the relationship by applying concepts to the diagram. However, they gave the incorrect response to Subtask 3.2 owing to substitution and computational errors, leading to the incorrect final answer.

Group 3 approached Task 1 by reflecting on the details learned during the initial intervention stage. They understood the task requirements and organised their thoughts to create an accurate diagram. They reached the correct conclusion for Task 1 by correctly applying trigonometric theorems. Furthermore, in Task 2, they applied their modelling skills, presenting a clear diagram and using it to calculate the lengths of AD and BD regarding y, α and Θ. However, when attempting Task 3, the group struggled to apply the correct solution method, resulting in an incorrect equation, as illustrated in the following examples (see Equation 3 and 4):

$$90° + \alpha + < DEF = 180° \Rightarrow < DEF = 180 - 135 = 45 \quad \text{[Eqn 3]}$$

$$tan45 = \frac{y - 250}{l} \Rightarrow l\,tan\,45 = y - 250 \Rightarrow l = y - 250 \quad \text{[Eqn 4]}$$

They considered another triangle from the figure and came out with the result of $tan45 = \frac{l}{250} \Rightarrow 250\,tan\,45 = y - 250 \Rightarrow y = 500m.$ The solution was presented inaccurately, and the trigonometric modelling principles and theorems were improperly exploited.

Despite using the correct theorem of the Sine rule, Group 4 had difficulty drawing the model. This affected their solution and made the final answer to Task 1 inaccurate. Group 4 attempted to draw the diagram for Task 2. However, they failed to identify the measurements in the diagram, which resulted in incorrect answers. Group 4 did not even attempt Task 3.

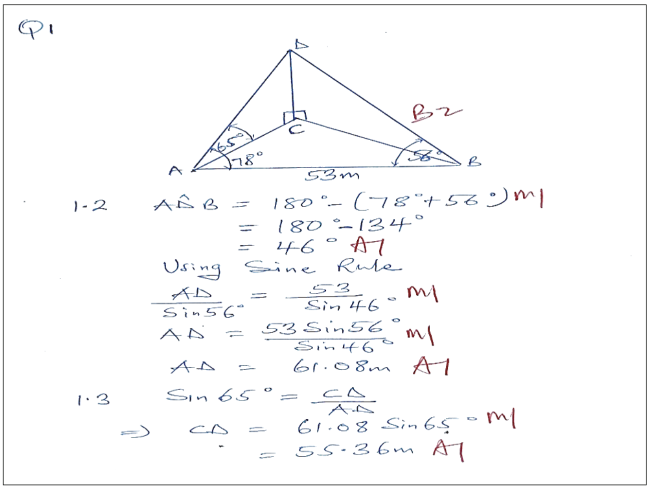
Group 5 thoroughly understood Task 1 and successfully transitioned the task's context from a real-world scenario to a mathematical model through mathematisation. They accurately estimated the necessary measurements in the model to find the correct solution. The group members reflected on relevant mathematical concepts, facts, formulas, and theorems critical to solving the task. Specifically, the pre-service mathematics teachers applied the concept that the sum of interior angles in a triangle is 180° to determine the 46° angle. They then used the Sine rule to arrive at the correct answers for subtasks 1.2 and 1.3, as illustrated in Figure 8.

Comparable concepts and formulas learned earlier during the intervention were applied by Group 5 to construct the

Task 2 model, which was then used to answer the subtask and obtain the correct answers. None of the group members could complete Task 3.

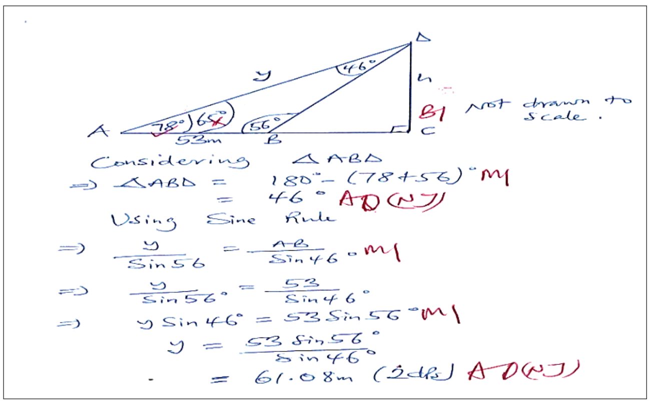
Group 6 utilised reflective thinking skills to analyse the tasks and deconstruct the data to create a model. After reviewing the task and recalling relevant facts, they developed a mathematical model based on an actual scenario. The group reconsidered mathematical theories, facts, formulas and concepts and applied the Sine rule to arrive at correct solutions. They completed subtasks 3.1 and 3.2 of Task 3 using the diagram. Additionally, they simplified their presentation of findings and solutions, making them accessible and relatable to concepts that Junior High School students might use.

Group 7 approached the task by reflecting on the intervention process and previously taught concepts, allowing them to understand the task thoroughly. They adopted a different approach to drawing the model, while partially accurate, was informed by their reading and comprehension of the task. Through extensive reflection, they calculated the length of AD as 61.08 m, although this approach was not entirely judicious because of the placement of two angles at angle CAD, as shown in Figure 9.



Source: Pre-service mathematics teachers sample response

FIGURE 8: Experimental Group 5 sample post-test solution.



Source: Pre-service mathematics teachers sample response

FIGURE 9: Experimental Group 7 sample post-test solution.

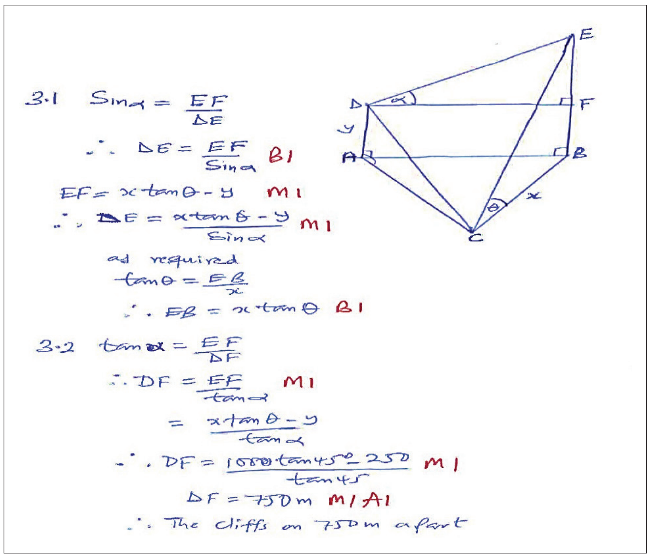
Their reflections involved retrieving information from memory, reviewing the task from simple to complex, investigating ideas, and applying concept development using the modelling process, including applying the Sine rule. Group 7 clarified and comprehended Task 2, producing a sophisticated model compared with that of Task 1. In addition, Group 7 demonstrated a significant level of reflection throughout the process.

Group 7 drew the Task 2 diagram with two different angles, 78° and 65°, at point A, which was geometrically impossible. As a result, they were awarded marks for their method but received no credit for the final answers, even though the answers were correct, because of the flawed approach. This group did not attempt Task 3, as they felt they lacked sufficient time.

In contrast, Group 5 effectively utilised their reflective thinking skills by analysing Task 3 and applying various theories. They accurately interpreted the given diagram and arrived at the correct answers, as illustrated in Figure 10.

Group C demonstrated creative thinking by generating ideas applied to design the diagrams for Tasks 1 and 2, resulting in correct answers through a focus on conventional concepts. For each subtask in Task 3, the group completed the tasks by analysing the diagram given. However, instead of deriving the required relationship, they merely substituted the provided measurements into the equation and solved it. This approach was not ideal for addressing the subtasks. Moreover, they had difficulty recalling basic concepts, and simplification strategies hindered their ability to model the given relationship accurately and arrive at correct answers.

Figure 11 presents Group C's post-test solutions for various tasks. The diagram showcases their approach to solving Tasks 1.1, 1.2, 1.3, 2.1, 2.2, 2.3 and 3.1. The annotations and



Source: Pre-service mathematics teachers sample response

FIGURE 10: Experimental Group 5 sample post-test result.

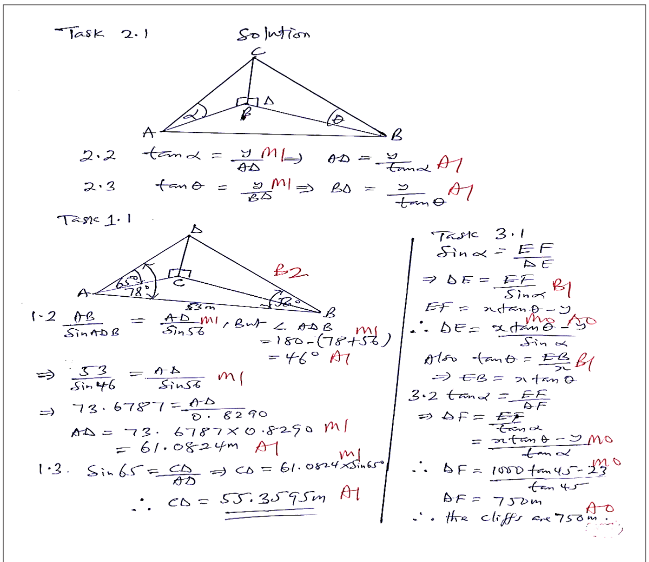
calculations illustrate the group's application of trigonometric concepts and their transition from theoretical understanding to practical problem-solving. Group C demonstrated their knowledge of concepts, such as the Sine rule, angle sum properties, and the relationships between angles and sides in right-angled triangles. The figure indicates the group's ability to connect abstract mathematical principles to real-world scenarios.

Figure 11 highlights the group's systematic steps in deriving lengths and angles, emphasising the accuracy of their mathematical reasoning. However, the figure also shows areas where the group's understanding or application of trigonometric concepts may have been inadequate.

These shortcomings may have stemmed from a misinterpretation of the task requirements, incorrect assumptions or minor errors in calculation that could have impacted the final results. These inadequacies highlight the importance of precision and the need to carefully verify each step in mathematical problem-solving.

Comparison Group G struggled to apply reflective thinking in recalling the necessary concepts and theories to analyse the given diagram and answer the subtask. Instead of systematically approaching the problem by leveraging their prior knowledge, they attempted to generate triangles from the overall diagram. However, this approach was insufficient, leading to an incorrect solution for the subtask, as illustrated in Figure 12.

Their inability to apply relevant trigonometric concepts, such as identifying and using angles and sides in the diagram, suggested a gap in their understanding or a lack of confidence in their problem-solving ability. This shortfall resulted in a fragmented approach, with the group attempting to break down the problem without a clear strategy and failing to solve the problem.



Source: Pre-service mathematics teachers sample response

FIGURE 11: Comparison Group C sample post-test solution.

Figure 12 highlights the challenges and emphasises the importance of reflective thinking and a foundational understanding of mathematical concepts in tackling complex problems. The figure underscores the need for students to connect theory with practice and to apply their knowledge to arrive at correct solutions methodically.

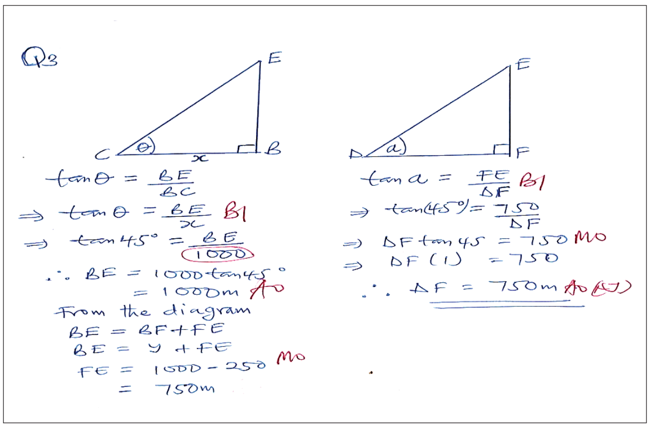
Figure 12 shows that Group G demonstrated its ability to produce correct diagrams derived from the central diagram and accurately applied trigonometric ratios. Specifically, they correctly identified and calculated tanα using the given angles and sides in the triangles. This indicated their understanding of basic trigonometric principles and their application to the problem. However, despite this correct initial step, the group encountered difficulties proceeding with the proper method to solve and prove the given relation. After calculating the tangent values, Group G struggled to connect these results with the required final solution. They could not utilise the trigonometric properties to advance their solution, leading to an incorrect final answer.

Interview findings: Comparison group

During interviews, the comparison group revealed insights into the effectiveness of the conventional approach following the intervention. These pre-service mathematics teachers found applying the appropriate theorem and trigonometric ratios challenging. They acknowledged their prior knowledge but failed to reflect deeply enough to retain it for real-world application. Their justifications for their techniques were often ineffective, leading them to repeatedly acquire, unlearn and relearn the rules and theorems needed to complete the tasks.

While most pre-service mathematics teachers could identify the concepts behind the Sine and Cosine rules, they also recognised the challenge of drawing accurate mathematical models or diagrams. Many students struggled to complete tasks because of insufficient understanding of the theory or its application.

The following sample from the interview between the modelling instructor and a pre-service mathematics teacher highlights several challenges:



Source: Pre-service mathematics teachers sample response

FIGURE 12: Comparison Group G sample post-test solution.

> 'What kind of mathematical modelling experience do you have?' (Modelling teacher educator)
>
> 'I never realised that I was studying mathematical modelling, even though I did understand some of the basic concepts at one point.' (Pre-service mathematics teacher)
>
> 'Do your tutors discuss or employ mathematical modelling in their teaching?' (Modelling teacher educator)
>
> 'No, the term 'mathematical modelling' has never been mentioned.' (Pre-service mathematics teacher)

At times, the modellling teacher educator had to explain the concept of mathematical modelling to respondents before they could answer the interview questions. For example, when the modelling instructor asked, 'Do you believe that mathematical modelling will have a significant impact on Basic School pupils?', one of the pre-service teachers responded:

> 'Yes, I believe it will have a significant impact due to the increased freedom for students to research and expand their knowledge. They won't feel that there's only one correct way to solve a problem or that maths is inherently difficult. Students might prefer real-life problems over traditional mathematical problems, as many tutors and pre-service teachers use modelling without realising it's mathematical modelling. With increased awareness, they would be better equipped to help basic school students understand mathematical concepts by teaching through mathematical modelling rather than relying on teacher-centred approaches.' (Pre-service mathematics teacher)

The interview responses from the comparison group revealed pre-service mathematics teachers' challenges in applying the conventional approach after the intervention. Despite acknowledging their prior knowledge, these teachers struggled to apply the appropriate theorems and trigonometric ratios. A lack of deep reflection hindered their ability to retain and use this knowledge in real-world contexts, leading to repeated cycles of acquiring, unlearning and relearning the necessary rules and theorems.

While many could identify the concepts behind the Sine and Cosine rules, they found it difficult to accurately draw mathematical models or diagrams, impacting their ability to complete tasks. This reflected a gap in their understanding of both theory and its practical application.

One pre-service teacher admitted to not realising they were studying mathematical modelling, indicating a disconnect between their learning experiences and recognising these experiences as part of a modelling framework. In addition, the term 'mathematical modelling' was reportedly never mentioned in their coursework, suggesting a lack of emphasis on this aspect of education.

The modelling teacher educator often had to explain mathematical modelling during the interview, showing a broader awareness issue. However, there was recognition of the impact of mathematical modelling on students, particularly in promoting independent thinking and a deeper understanding of concepts.

The responses suggested that the conventional approach may fail to foster deep reflection, practical application of theoretical knowledge and integration of mathematical modelling in teaching. This finding highlights the need for focussed training to prepare pre-service teachers.

Interview findings: Experimental group

Pre-service mathematics teachers could generally solve the tasks with relative ease and recognise the importance of the intervention. One group drew the model correctly but struggled to apply the appropriate theorems to arrive at the correct answers. However, most pre-service teachers successfully interpreted and validated their results to ensure accuracy.

Although some group members mentioned the challenges they faced in drawing the diagrams, most effectively utilised the modelling framework to produce accurate results. Below is a sample of an interview:

> 'What kind of mathematical modelling experience do you have?' (Modelling teacher educator)
>
> 'Yes, during our internal quiz, we were asked to estimate the number of hours and the amount of money that needed to be paid in Ghanaian cedis. For example, if someone was paid for working six days, how much would they be paid? Similarly, if someone worked seven days, how much would they be paid? When calculating the total, which option would be best?' (Pre-service mathematics teacher)
>
> 'Do your tutors discuss or employ mathematical modelling in their teaching?' (Modelling teacher educator)
>
> 'Yes, in the Algebraic Thinking course, linear equations were modelled using algebraic tiles.' (Pre-service mathematics teacher)
>
> 'Have you dealt with the New Common Core Mathematics Curriculum (NCCMC) statement's aspects of mathematical models? If so, could you please explain how?' (Modelling teacher educator)
>
> 'Yes, I taught Junior High School mathematics during my macro-teaching [*off-campus*] experience. Although many tutors focus on teaching for learning, which is not the best approach, I was not impressed with how the students were thinking. This modelling approach would help students think more deeply. Allowing students to conduct independent research, explore, and take ownership of their education would significantly improve their comprehension of mathematical concepts.' (Pre-service mathematics teacher)

The interview responses provided insights into the experiences of pre-service mathematics teachers with mathematical modelling in their education and teaching. The pre-service teacher recounted a quiz where they estimated payments for different working days, demonstrating some practical experience with mathematical modelling. However, it was more focussed on basic arithmetic or algebra rather than a deeper engagement with modelling. They also mentioned that mathematical modelling had been included in their coursework, particularly in an algebraic thinking course where linear equations were modelled using algebraic tiles. However, this exposure seemed limited to specific courses rather than a widespread approach to their education.

Reflecting on their experience with the New Common Core Mathematics Curriculum (NCCMC) during macro-teaching, the interviewee expressed concern that many tutors emphasised 'teaching for learning', which the teacher felt was ineffective. The interviewee believed mathematical modelling could foster more profound, more critical thinking among students. The pre-service teacher advocated for a student-centred approach, where learners could explore and take ownership of their education through modelling.

The teacher also expressed dissatisfaction with students' thinking during their teaching experience, suggesting that conventional methods were not promoting deep understanding. The interviewee believed integrating mathematical modelling into teaching could help students approach problems from multiple angles and think critically.

The responses suggested that while pre-service teachers experienced little exposure to mathematical modelling, they saw its potential to enhance student learning and critical thinking. The interviewee advocated for the integrated use of modelling in teaching, moving beyond surface-level understanding, engaging students deeply and acknowledging that their experiences with modelling may have been limited and that there was room for further development in this area.

Discussion

The pre-test findings revealed that the experimental group (Subgroups 1, 2, 3 and 5) struggled with Task 3, successfully completing only Tasks 1 and 2. However, Groups 4 and 7 could not translate the real-world model into a mathematical model, drawing an appropriate diagram and aligning with the theories proposed by Csíkos, Szitanyi and Kelemen (2012). After thoroughly examining, verifying and validating their results, they used the diagram from Task 3 to obtain correct results, even though these differed from the ideal model.

In the comparison group, Group C divided the task into right-angled triangles and solved it to obtain the correct answer. However, Groups A, B, C, D and F could not complete Tasks 2 and 3. Group G, on the other hand, applied their knowledge creatively but struggled with drawing or finalising diagrams, affecting their performance in the pre-test tasks. As Rellensmann et al. (2017) suggest, failing to sketch or finalise diagrams hinders understanding mathematical modelling.

The pre-test results indicated that the comparison group did not use reflective thinking to uncover concepts and complete tasks. Consequently, many group members found applying their skills to the modelling tasks challenging and failed to attempt most mathematical problems. Rellensmann et al. (2020) recommend that students repeat procedures by sketching diagrams if they fail to reach a solution. The limited professional knowledge or lack of understanding among pre-service teachers became a significant barrier, consistent with findings by Breiner et al. (2012) and Corlu and Capraro (2014).

The findings highlighted that pre-service mathematics teachers had limited knowledge of mathematical modelling, aligning with Borromeo-Ferri's (2018) finding. Many solved the questions without utilising reflective thinking, which aligns with Galbraith's (2012) finding that pre-service teachers focussed on converting real-world problems into mathematical terms without fully engaging in deeper reflective thinking about the broader context of the problem. They were essentially applying mathematical techniques to real-world scenarios without critically reflecting on the underlying concepts or the significance of the problem-solving process.

Post-test results showed that the experimental group performed better in modelling problems than the comparison group, which is consistent with the findings of Yasa and Karatas (2018), who indicated that their experimental group outperformed the control group in mathematical modelling. Furthermore, in this study, the pre-service mathematics teachers in the experimental group demonstrated competencies such as knowledge about modelling problems, classroom management during modelling activities, and the ability to interpret and respond to students' thinking, consistent with Blum's (2011) and Schmidt's (2011) findings. Groups 1, 2, 6 and 7 in the experimental group validated their solutions through reflective thinking, while Group 5 struggled with Task 3, and Group 4 failed to complete any tasks despite their efforts.

The primary finding was that the experimental group used reflective thinking in post-test modelling. To address the tasks, they employed flexible approaches, such as problem-based learning and inductive-deductive reasoning. In contrast, the comparison group fell short in these areas., to solve mathematical problems and reach conclusions. These approaches aligned with Lu and Kaiser's (2022) assertion that adaptable strategies lead to successful mathematical outcomes.

Reflective thinking, often seen as a psychological phenomenon, is realistic when viewed as a pedagogical concept (Clarà 2015). Reflection plays a central role in teacher education and professional development, as noticed by Agustan et al. (2017), Amidu (2012) and Lim (2011). Interpreting a problem, unpacking information, modelling a real scenario, mathematising it into a mathematical model and using flexible techniques to arrive at solutions requires reflective thinking.

The experimental group demonstrated more reflective thinking than the comparison group, aligning with the findings of Thahir et al. (2019) and Agustan et al. (2017), who emphasise that reflective thinking helps identify ideas, concepts, formulas and theorems needed to solve mathematical problems using the modelling approach. The analysis of covariance indicated a statistically significant difference between the two groups, with a large effect size of 0.33, showing a significant impact of the mathematical modelling process on the reflective thinking ability of pre-service mathematics teachers. This result aligned with Salha and Qatanani's (2021) findings.

One of the essential skills or competencies pre-service mathematics teachers should develop is a content-oriented approach. According to Gocheva-Ilieva et al. (2018), a content-oriented approach involves focussing on the deep understanding of mathematical content and concepts, ensuring that teachers are not just proficient in procedural knowledge (how to solve problems) but also in conceptual understanding (why the solutions work and how they apply to various contexts).

In this study context, the experimental group benefited from an intervention that utilised a theoretical framework designed to enhance their modelling competencies. This framework included strategies to help pre-service teachers understand mathematical concepts and apply them in real-world scenarios. By linking these modelling competencies with a pedagogical mathematics activity framework, the intervention ensured that pre-service teachers were not just learning theory in isolation but also developing the ability to teach concepts.

This holistic approach of combining content knowledge with pedagogical skills led to the experimental group performing better than the comparison group. The superior performance is visually represented in the side-by-side boxplot in Figure 6, showing the distribution of scores or outcomes for both groups and highlighting the positive impact of the intervention on the experimental group. The boxplot results showed that the experimental group, outperformed the comparison group, which may not have received the same level of comprehensive training, thanks to the integrated approach of the intervention.

Conclusion

Pre-service mathematics teachers in Ghana should be trained to move beyond solving word problems and engage with real-life situations or global events through mathematical modelling. As Tan and Ang (2012) and Durandt (2021) suggested, pre-service teachers must be helped to tackle simple to complex authentic problems and bridge the gap between everyday mathematical language and educational discourse. To achieve this, mathematics pedagogy must be grounded in the pre-service teachers' prior knowledge and enhanced through reflective thinking skills. This approach involves incorporating horizontal and vertical mathematisation into mathematics lessons, improving the relevance and depth of the subject for pre-service mathematics teachers.

Horizontal mathematisation helps to translate real-world problems into mathematical language, making the subject applicable to everyday situations. Vertical mathematisation deepens understanding by connecting and abstracting mathematical concepts. By integrating both, pre-service teachers would develop a comprehensive skill set, enabling them to teach mathematics as both a practical tool and an interconnected field of knowledge. This dual focus makes mathematics meaningful and equips future teachers to approach problems from multiple perspectives.

Given the positive impact observed from the intervention using the modelling approach, the researchers propose the establishment of a mathematical modelling laboratory equipped with modelling-eliciting materials at the Colleges of Education in Ghana. Additionally, it is recommended that mathematical modelling should become a core competency within the mathematics curriculum at Ghana's Colleges and Basic Education levels.

Recommendation for future research

Future research can investigate how college tutors apply mathematical modelling to pre-service teachers and implement these strategies at the Basic Education level in Ghana. In addition, studies could include interviews and observations of tutors to examine how mathematical modelling is being integrated into the curriculum at Colleges of Education in Ghana.

Longitudinal studies could track pre-service teachers' progress as they transition into in-service teachers, assessing how their training in mathematical modelling influences their teaching practices and their students' learning outcomes over time.

In terms of curriculum development, future research could focus on designing and assessing specific curricular materials or teaching strategies that integrate mathematical modelling, creating new modules or resources tailored to the needs of pre-service teachers in Ghana.

Technology integration is another crucial area for future research. Studies could examine how digital tools and technology can support the teaching and learning of mathematical modelling, enhancing engagement and understanding among pre-service teachers.

By pursuing these research avenues, a comprehensive understanding of how mathematical modelling can be effectively integrated into teacher education could be developed, ultimately improving mathematics education in Ghana.

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Authors' contributions

H.I.B. contributed to the literature review, methodology, data collection, analysis and writing of the original draft of the article. R.G. contributed to the conceptualisation of the study, resources, writing, reviewing and editing the final draft of the article.

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Data availability

The data that support the findings of this study are available from the corresponding author, H.I.B. upon reasonable request.

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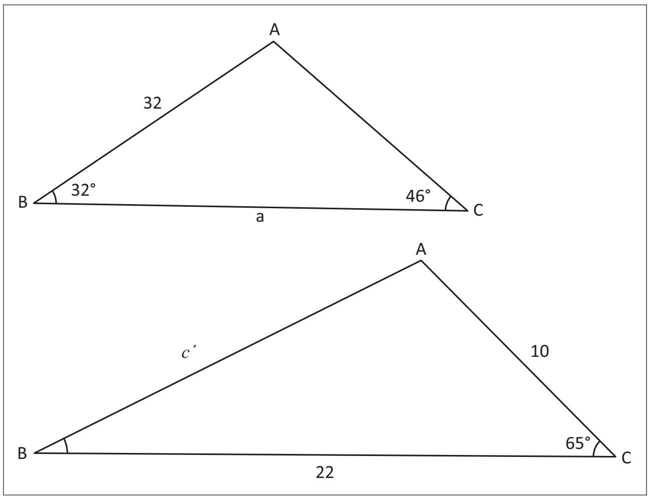
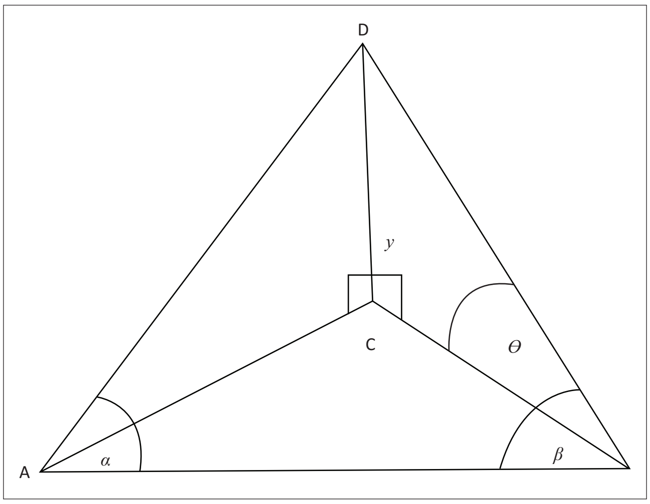
Appendices starts on the next page →

Appendix 1: Data collection instrument (pre-test)

1. Determine the value of the unknown variable in each of the following triangles, correct to 1 decimal place:



2. In the diagram below, D is a point vertically above C. DC is y-metre long. The angle of elevation of D from B is θ. Angle DAB = α and DÉA = β.
   2.1 Determine the length of DB regarding y and θ.
   2.2 Show that $AB = \frac{y\,Sin(\alpha+\beta)}{Sin\theta\;Sin\alpha}$



3. A spotlight for a theatre production illuminates a triangular area on stage. Actors are to stand at the corners of the illuminated area at P, Q and R. The actors at P and R have to stand 5 m and 4 m away from the actors at Q, respectively. The angle of elevation of S from P is 45°, and the angle of elevation of S from R is 60° if the spotlight is placed at a point vertically above PR.
   3.1 Draw a diagram to illustrate the above information
   3.2 Determine how the actors at P and R must stand from each other
   3.3 The actor at P enters on stage by sliding down a wire from S to P. How long is the wire that the actor slides along?

Appendix 2: Data collection instrument (post-test)

1. A, B and C are three points in the same horizontal plane, and AB is 53 m long. CD is a vertical tower, and the angle of elevation of D from A is 65°. < DAB = 78° and < DBA = 56°
   1.1 Draw a diagram to illustrate the above information
   1.2 Determine the length of AD
   1.3 Determine the height of the tower CD
2. In a diagram, C is a point vertically above D. CD is y metres long. The angle of elevation of C from B is θ, and the angle of elevation of C from A is α. Angle ADB = β
   2.1 Draw a diagram to illustrate the given information
   2.2 Determine the length of AD regarding y and α
   2.3 Determine the length of BD regarding y and θ
3. A telephone cable is to be erected between 2 Cliff sides, AD and BE. An engineer stands at point C in the same horizontal plane as the foot of the cliffs. He measures the angle of E from C and D to θ and α, respectively. Cliff DA is y meters in height, and C is x metres from the foot of cliff BE.
   3.1 Show that the length of the telephone cable is $\frac{x \tan\theta - y}{Sin\,\alpha}$
   3.2 If $x = 1000m, y = 250m$ and $\theta = \alpha = 45°$. What is the distance between the Cliffs?