African Journal of Teacher Education and Development
ISSN: (Online) 2958-0986, (Print) 2958-8650

Error analysis in fraction addition and subtraction using structured observed learning outcomes

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Dates:
Received: 07 Mar. 2024
Accepted: 16 May 2024
Published: 27 June 2024

How to cite this article:
Dube, A. & Maseko, J.S., 2024, 'Error analysis in fraction addition and subtraction using structured observed learning outcomes', *African Journal of Teacher Education and Development* 3(1), a53. https://doi.org/10.4102/ajoted.v3i1.53

Background: Fractions are important because of their critical role in more advanced mathematics. While they are central to mathematics learning, they pose a challenge to learners. The use of the structured observed learning outcomes (SOLO) taxonomy attempts to assist teachers to analyse the learners' errors.

Aim: This study reports on analysing Grade 8 learners' errors in the addition and subtraction of fractions using the SOLO taxonomy.

Setting: The study focusses on the errors made by learners when solving problems on the addition and subtraction of fractions. Participants were drawn from Grade 8 learners ($N = 115$) in a school in inner Johannesburg city.

Methods: Qualitative and quantitative data were collected from a paper and pencil test on addition and subtraction of fractions given to learners to solve. The learners' errors were analysed using the SOLO taxonomy framework.

Results: Findings indicate that learners made errors in the addition and subtraction of fractions that were classified in the different levels of the SOLO taxonomy, ranging from the unistructural to the extended abstract.

Conclusion: The study concludes that it is crucial to analyse the learners' errors when solving addition and subtraction of fractions to determine their understanding of working with fractions and using the SOLO taxonomy framework can effectively map the learners' specific level of working with fractions.

Contribution: The study's results will equip in-service teachers, student teachers and teacher trainers with knowledge of learners' different levels of errors when dealing with addition and subtraction of fractions to help learners overcome these errors.

Keywords: SOLO taxonomy; addition; subtraction; fractions; pre-structural; unistructural; multi-structural.

Introduction

The learners' performance in mathematics in South Africa has generally been poor over the years. Lovemore, Robertson and Graven (2021) found that mathematics education is a worrying issue in South Africa. They further pointed out that teaching and learning fractions, in particular, is one of the key challenges teachers and learners face. Patel (2018) observed that in diagnostic tests conducted by a maths tutoring service, Bright Futures, in 2017 and 2018, the learners' performance at a Grade 4–7 level on fractions was just 31%. In this study, we decided to investigate the errors made by Grade 8 learners in the addition and subtraction of fractions in the form of a question: 'What competency level do these Grade 8 learners bring to the class in terms of working with fractions?' We mapped the assessment items and analysed the errors using the structured observed learning outcomes (SOLO) taxonomy framework. This taxonomy, the researchers believe, would facilitate a clearer path to responding to and supporting learners, students and teacher-development practitioners. The mapping of errors according to the levels would highlight the focus on teaching and closing gaps where necessary. We argue that when communication flows between teacher trainers, pre-service teachers and in-service teachers without any obstacles, there is a better chance to arrest the perpetually limited knowledge of dealing with addition and subtraction of fractions.

Teacher trainers assume the students who enrol for the teacher training programmes have sufficient knowledge of working with fractions. Maseko, Luneta and Long (2019) report that the first-year students in this study had minimal knowledge of working with fractions. Of the 117 students, only 17% got the correct answer, 71% were totally incorrect and 12% did not answer the questions about addition (three) and subtraction (three) of fractions.

Challenges of teaching and learning fractions

A good understanding of fractions entails that learners observe that several attributes that apply to whole numbers do not apply to fractions. According to Siegler et al. (2013), learners make two main types of errors in symbolic fractional problems. These are conceptual errors and procedural errors. Bruce et al. (2013) also identified teacher-related errors. These types of errors are discussed in the following sections.

Conceptual errors

Independent whole number errors: Independent whole number errors involve performing the mathematical operation independently on numerators and denominators (Alkhateeb 2019; Aksoy & Yazlik 2017; Braithwaite & Siegler 2021, 2024; Ni & Zhou 2005; Siegler & Lortie-Forgues 2015, 2017; Tian & Siegler 2017; Vamvakoussi, Van Dooren & Verschaffel 2012; Vamvakoussi & Vosniadou 2004), for example, $\left(\frac{1}{2}+\frac{1}{3}=\frac{2}{5}\right)$. Jigyel and Afamasaga-Fuata'I (2007) concurred with Siegler et al. (2013). They averred that some learners often view a fraction as two separate whole numbers and, then use whole number reasoning when dealing with fractions. To emphasise this, Pant (2019) argued that learners consider a fraction as a separate entity (in the form of or made up of two numbers) and perceive the fraction as an object disconnected from the number system. As such, they expect fractions to behave like natural numbers. Research shows that this misconception is also found in college students. Research conducted by Lee and Boyadzhiev (2020) on the understanding and misconceptions of fractions by underprepared college students found that less than half of the 22 students in the study failed to compute the problem $\frac{2}{3}-\frac{4}{8}$ correctly.

When moving from whole number thinking to dealing with fractions, learners need a good conceptualisation of various interpretations of fractions. Without this knowledge, learners will find it challenging to understand the possible meanings of a numerator, a denominator, the whole number in a mixed fraction and the distinctions between them (Jigyel & Afamasaga-Fuata'I 2007; Petit et al. 2010). For example, when we consider a fraction representation as a part–whole relationship, the number at the top (numerator) represents the number of parts of interest in the whole and the number at the bottom (denominator) represents the number of equal parts in the whole. When working with a fraction as a quotient, the numerator is a quantity (dividend) divided by the denominator (divisor). According to Bruce et al. (2013), learners must understand the different roles played by the numerator and denominator and that the interpretations vary depending on the role.

Gap thinking: Pearn and Stephens (2004) found that 'gap thinking' is another common but inappropriate way of thinking about fractions exhibited by learners. For example, a learner who is comparing three-quarters and four-fifths may think that the two fractions are equal. The learner's reasoning here is that three-quarters is one part short of making a whole and also four-fifths is one part short of making a whole. The learner has observed that the gap between 3 and 4 in the first fraction and 4 and 5 in the second fraction is one and thus considered the numerical difference and not the actual size of the parts. The learner must be able to compare the magnitude of the named fractional amount to the whole.

Fraction magnitude misconception: The fraction magnitude misconception is closely related to the 'gap thinking' error. Tian and Siegler (2017:615) argued that 'Accurate magnitude knowledge can help students evaluate the plausibility of answers to arithmetic problems and reject procedures that lead to implausible answers (e.g. $\left(\frac{1}{2}\times\frac{2}{2}=\frac{2}{4}\right)$. The difficulty in processing symbolic magnitudes of fractions does not affect only learners but also expert mathematicians (Obersteiner et al. 2013). This puts teachers in the spotlight, as confirmed by Namkung and Fuchs (2019) when they found that 'many elementary school teachers lack fraction competence with ordering fractions, adding fractions, and explaining computations for fractions' (p. 38). An example of fraction magnitude misconception is when a learner considers $\frac{1}{4}$ as greater than $\frac{1}{3}$ because denominator 4 is greater than denominator 3.

Procedural errors

Operation errors: The wrong fraction operation errors involve using correct components for another fraction arithmetic operation on an operation where they are incorrect. A common example involves maintaining common denominators in multiplication problems, as is appropriate in addition and subtraction problems, for example $\left(\frac{1}{2}\times\frac{2}{2}=\frac{2}{4}\right)$. These errors indicate a lack of understanding of the conceptual basis of fraction mathematical procedures. Siegler et al. (2013) further noticed that these findings suggest that many children's knowledge of fractions includes a mix of correct procedures, components of procedures detached from the relevant mathematical operation and whole number mathematical procedures. In some cases, learners tend to generalise previously learnt procedures to new situations where they do not apply (Makonye 2011), which causes them (learners) to make errors. The authors argue that the errors made by learners in addition and subtraction of fractions lead to errors at a higher level in Calculus such as equation-balancing errors and pseudo-linearity errors as found by Hirst (2002).

The effect of operations: Some challenges arise when learners lack an understanding of the effect of operations on fractions. Siegler and Lortie-Forgues (2015) called this 'the direction of effects error'. In all positive numbers addition results in an answer greater than each of the numbers being added. In subtraction, the answer is less than the subtrahend. The direction of effects is the same in both operations. In multiplication and division, the direction of effects is not the same as in addition and subtraction. It depends on the size of the numbers involved. If one is multiplying numbers between zero and one the answer is less than any of the numbers being multiplied and when one is dividing by a number between zero and one the result is a number greater than the dividend. Siegler and Lortie-Forgues (2015) argued that this conceptual knowledge has not been systematically extended to fractions; hence, learners may fail to rationalise the results of the different operations on common fractions.

Looking at the challenges discussed above, two components emerge. The challenges fall under conceptual (the knowledge, ideas and their relationships that help one to carry out procedures of solving problems) and procedural knowledge (the actions or rules used to solve problems on fractions).

Teacher-related errors

Other challenges are observed from the teaching angle. Studies have shown that in some cases, teachers lack an understanding of fractions (Depaepe et al. 2015; Garet et al. 2010). Pant (2019) argued that the quality of teaching and the teacher's knowledge play pivotal roles in minimising the difficulties faced by learners. Cortina, Visnovska and Zuniga (2014) pointed out that the strategies teachers use in teaching and learning mathematical concepts may become obstacles to teaching and learning complex concepts in the future. They further explained that teaching and learning challenges arise when metaphors, representations and other instructional resources used by teachers result in learners' ideas that are inconsistent with more complex mathematical learning goals. Two examples of problematic teacher approaches are discussed here.

Imprecise language: Bruce et al. (2013) pointed out that confusion about the role of the numerator and denominator arises from the inadvertent use of imprecise language. Describing two-thirds as 'two over three' or 'two out of three' leads to learners conceptualising each as a separate whole number rather than recognising the multiplicative relationship that is inherent in the notation (that is to say that two-thirds is two one-third units or that it is referencing two one-thirds of a whole (Mack 1995).

Procedural knowledge thrust: Pant (2019) also found that 'The thrust of the teachers generally remains in imparting procedural knowledge and, less, or no emphasis is given to the conceptual understanding of fractions' (p. 20). This explains the learners' lack of foundational understanding of the meaning and ways of thinking about a fraction. Without the requisite conceptual understanding, such as the importance of equivalence, estimation, unit fractions and part–whole relationships, learners struggle to complete calculations with fractions.

It is, therefore, important, as Gabriel et al. (2013) emphasised, for the teacher to understand these challenges in teaching and learning fractions to ameliorate learners' challenges in the conceptual and procedural understanding of fractions.

The structured observed learning outcomes taxonomy

Ball (1990) emphasises the use of instructional representations in the effective teaching of fraction concepts. This lays the foundation for higher-level topics in mathematics such as Algebra and Calculus. The authors believe the SOLO taxonomy can be used to determine the learners' levels of understanding fraction concepts from the pre-structural to the extended abstract levels. Likewise, it can be used to determine the learners' understanding of solving calculations involving fractions. In this study, the SOLO taxonomy assisted the researchers in making sense of the learners' levels of understanding of the addition and subtraction of fractions. Notwithstanding that there are other models of measuring learning outcomes, for example, Bloom's taxonomy, the van Hiele model and the Reflective thinking model, the researchers opted for the SOLO model. Chan et al. (2002), in their comparative study on the SOLO taxonomy, Bloom's taxonomy and the Reflective thinking model, found that the SOLO taxonomy is ideal for measuring the learning outcomes for different subjects. Adeniji, Baker and Schmude (2022) systematically reviewed studies on the SOLO model and mathematics education. In a sample of 62 articles, their findings indicated that the SOLO model:

> [*A*]ppropriately reflects students' learning outcomes; there is a direct relationship between students' performances and their SOLO levels; and the SOLO model could explain several other developmental theories and contribute to the development of mathematics curricula. (p. 1)

Several researchers consider the model comprehensive and that it provides an objective window for classifying the learners' levels of understanding the various concepts (Chick 1998; Lake 1999; Xistouri 2007). It was because of these benefits that the researchers opted for this model.

A brief history of the structured observed learning outcomes taxonomy

The SOLO taxonomy was founded by Biggs and Collis (1982). The model was born out of their concern that the traditional methods of assessment of the learners' performance were deficient in valuable information on the learning growth of the learners. They felt that the traditional assessment methods ignored the qualitative part of the performance. Biggs and Collis (1982) argued that the SOLO model addresses the confusion created by the inconsistency in matching the test scores with Piaget's developmental stages. The SOLO model consists of levels that aim to describe the learners' responses to test items, rather than the learners themselves. As Biggs and Collis (1982) pointed out 'they describe a particular performance at a particular time. They are not meant as labels to tag learners' (p. 23).

The levels of the structured observed learning outcomes taxonomy

The SOLO taxonomy is based on the premise that learning should aim at increasing knowledge (quantitatively) and deepening understanding (qualitatively) (Kusmaryono 2018). Biggs and Collis (1982) argued that the learning process needs to consider how much has been learnt and how well it has been learnt. They identified five levels that describe the learners' understanding of concepts in various subjects. The levels were succinctly summarised by Biggs (1996) as follows:

1. Pre-structural: The learner has not met the concept before and therefore fails to attack a given problem appropriately. No Grade 8 learner was expected to be at this level in addition and subtraction of fractions.
2. Unistructural: One or a few aspects of the task are picked up and used (understanding as nominal). We allocated equivalent fractions as the entry level, $\left(\text{e.g}\,\frac{x}{12}=\frac{14}{24}\right)$. The key to this was recognising the value and restrictions associated with the position of the unknown. This knowledge of equivalence needed to be applied to all the other question items.
3. Multistructural: Several aspects of the task are learned but are treated separately (understanding as knowing about). The learners needed to work with equivalence in addition and subtraction of fractions, $\left(\text{e.g}\,\frac{11}{12}=\frac{14}{24}\right)$. This level was used to analyse the learners' understanding of the addition and subtraction of simple fractions with related and unrelated denominators.
4. Relational: The components are integrated into a coherent whole, with each part contributing to the overall meaning (understanding and appreciating relationships). We made the mixed numbers as the next level of complexity. The learners needed to pull other knowledge bases of working with fractions (converting to improper fractions), $\left(\text{e.g}\,6\frac{1}{12}-3\frac{2}{3}-1\frac{1}{2}\right)$. This level focussed on the knowledge of addition and subtraction of mixed numbers involving two terms.
5. Extended abstract: The integrated whole at the relational level is reconceptualised at a higher level of abstraction, which enables generalisation to a new topic or area or is turned reflexively on oneself (understanding as far transfer and as involving metacognition) (p. 352). This level was allocated to double addition and subtraction of mixed fractions with more care on working with subtraction and the use of brackets for those who choose that approach, (e.g. Eqn-6) The extended abstract level was used to check the learners' knowledge of the addition and subtraction of mixed numbers with related and unrelated denominators involving three terms. This also extended to alternative methods of adding fractions (adding or subtracting whole numbers separately and then the fractions on their own).

The SOLO taxonomy levels define a hierarchical structure of the learners' understanding of concepts from simple to complex. Therefore, the model explains increasing complexity in concept understanding by analysing learners' task responses.

Application of the structured observed learning outcomes taxonomy

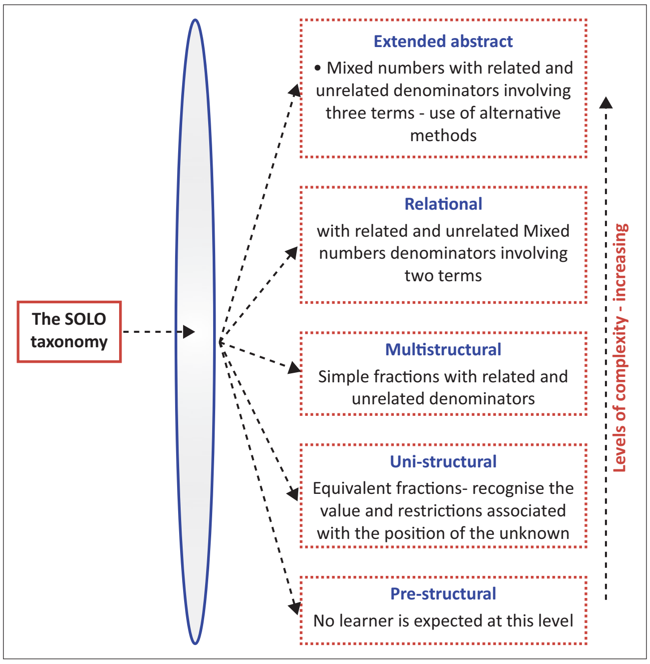
Various studies in the field of mathematics (mostly in Algebra, Symmetry, Problem-solving and Trigonometry) have utilised the SOLO taxonomy (Chick 1998; Christinove & Mampouw 2019; Kaharuddin & Hajeniati 2020; Lian & Yew 2012; Mulbar, Rahman & Ahmar 2017). A few known studies have focussed on applying the SOLO taxonomy, specifically fractions. For example, Whitehead and Walkowiak (2017) investigated the preservice elementary teachers' understanding of operations for multiplication and division of fractions while enrolled in a mathematics methods course. Mirnawati, Bernard and Asdar (2023) analysed Grade 7 learners' understanding of the material on fractions using the SOLO taxonomy.

Using the test item levels, the researchers described the learners' performance levels in addition and subtraction of fractions according to the SOLO model. The test was designed and mapped the items to the levels of complexity and cognitive demand: equivalent fractions (uni-structural), simple fractions with related and unrelated denominators (multistructural), mixed numbers with related and unrelated denominators involving two terms (relational) and mixed numbers with related and unrelated denominators involving three terms (extended abstract). These levels were aligned to the SOLO taxonomy structure to describe the learners' understanding of addition and subtraction of fractions with a type that increases the cognitive demand, as shown in Figure 1.

This demonstrates the increasing complexity of concepts on fractions as described in the SOLO taxonomy.

Research methods and design

A total of 115 learners participated in the study. The school has four streams of Grade 8 learners, totalling 115 learners.



SOLO, structured observed learning outcomes.

FIGURE 1: The application of the SOLO taxonomy in the study.

The classes are named 8L, 8M, 8R and 8S, with the letters having no specific significance except identification. The learners were given eight problems to solve, covering the following levels: Equivalent fractions, simple fractions with related and unrelated denominators, mixed numbers with related and unrelated denominators involving two terms, and mixed numbers with related and unrelated denominators involving three terms. Equivalent fractions were included in the test items because they provide pre-requisite knowledge for the addition and subtraction of fractions. The creation of the tools and scope was informed by the work on fractions covered in earlier grades leading to Grade 7. The Department of Basic Education (2011, 2020) retained the original section on common fractions in the trimmed and re-organised curriculum and this was designed as a catch up plan to address the impact of the coronavirus disease 2019 (COVID-19) pandemic on the attendance timetable (p. 17, p. 8). The skills they were supposed to have covered by the end of their revised plan were the sections that became the base of our selection of skills for the investigation. The range of work in fractions included equivalent fractions to mixed numbers (fractions) and the possible relationships between their denominators by the end of Grade 7 (DBE 2011:17). After solving each problem, each learner was asked to write down the steps they had followed to solve the problem. They were also asked to write their experiences during the solving process on the answer sheet. The researchers here expected to capture comments such as 'the problem was difficult for me, or I did not know how to go about solving the problem'. The answers to the test items helped the researchers check their conceptual and procedural understanding of fractions and identify any misconceptions the learners may have had about the addition and subtraction of fractions. The learners' answers and written comments were then analysed regarding errors they may have made. The results are presented in the 'Results' section.

Ethical considerations

Ethical clearance to conduct this study was obtained from the University of Johannesburg Faculty of Education Research Ethics Committee (reference no.: Sem 1-2021-132).

Results

The errors were classified into codes, as shown in Table 1.

Errors made in the test

Equivalent fractions: The error codes E1 to E4 were related to problems on equivalent fractions (see Table 2). They ranged from applying the difference between the numerator and denominator in the complete fraction to the fraction with a missing value, failure to use the cross-multiplication concept in solving the equivalent fractions problems, adding the numerator and the denominator of the complete fraction to multiplying the numerator and the denominator of the complete fraction. The following errors were noticed in the participants' work on questions (a) and (b).

In question (a), the participants made 45 errors. The E2 type of error was the most prevalent of the errors made. Fifty-two per cent of the errors made fell under this category. In the E2 error, participants failed to use the cross-multiplication concept to solve for the missing component of one of the pairs of equivalent fractions.

The second highest number of errors was found in types E3 and E4. These constituted 20% of each of the total errors in question (a). In error E3, learners added the numerator and the denominator of the complete fraction. For example, one learner added 2 and 3 in this problem: $\frac{2}{3} = \frac{x}{18}$ and got 5 as the missing component of the second fraction.

Addition of fractions: In addition to fractions, three types of problems were given to the participants. In question (c), learners were required to add common fractions involving two terms. In question (e), they were asked to add mixed numbers involving two terms with related denominators. In (g), they had to add mixed numbers involving three terms with unrelated denominators. The results of the errors made by the learners are shown in Table 3.

The most prevalent error in addition of simple fractions involving two terms (The denominators were unrelated in this task), in question (c), was the E5.1 which made 66% of the errors made. In this error, learners were adding the numerators together and then adding the denominators together. For example, in question (c) $\frac{3}{4}+\frac{2}{5}$, one learner's answer was $\frac{5}{9}$.

Error E6 formed 13% of the errors made in question (c). Learners making this error failed to work out the equivalent fractions once they had determined the lowest common multiple of the denominators of the fractions.

Only one E9-type error was made. The learner failed to compute properly. In this case, learner 8S10 added 8 and 13 and answered 103 instead of 23. Three learners submitted incomplete answers.

In question (e) $2\frac{3}{4}+2\frac{1}{2}$, the learners were given fractions with related denominators. Two approaches were used by the learners in solving or attempting to solve the problem. Some learners added the whole numbers independently and then the fractions on their own, while others changed or attempted to change the mixed numbers to improper fractions before adding. In either method, errors were noted. Sixty per cent of the learners made error E5.1. Learners treated the fraction components as independent whole numbers in this error category.

Fourteen per cent of the learners made an E6 error. E5.2 and E8 errors were made by 8% each of the participants. Error E5.2 entailed learners adding or subtracting the numerators and picking any one of the denominators, and in error E8, learners failed to change a mixed number to an improper fraction.

TABLE 1: Table displaying error codes, types and examples.

Error codes	Error types	Examples
Fraction equivalence		
El	Applying the difference between numerator and denominator in the complete fraction to the fraction with a missing value	The difference between 2 and 3 is 1 and therefore applying this idea in the other fraction will give an answer $\frac{17}{18}$
E2	Failure to use the cross-multiplication concept in solving the equivalent fractions problems	
E3	Adding the numerator and the denominator of the complete fraction	$\frac{2}{3}=\frac{5}{18}$
E4	Multiplying the numerator and the denominator of the complete fraction by same number	$\frac{4}{5}=\frac{20}{25}$
Addition and subtraction of fractions		
E5.1	Adding or *subtracting* numerators together and then adding or *subtracting* denominators together	$\frac{3}{4}\pm\frac{2}{5}=\frac{5}{9}\left[\frac{1}{-1}\right]$
E5.2	Adding or subtracting the numerators and picking any one of the denominators	$\frac{3}{4}\pm\frac{2}{5}=\frac{5}{9}\left[\frac{1}{5}\right]$
E5.3	Use of wrong operation e.g. Added or subtracted when supposed to subtract or add	
E5.4	Adding or subtracting the numerators and multiplying the denominators	$\frac{3}{4}\pm\frac{2}{5}=\frac{5}{20}\left[\frac{1}{20}\right]$
	Multiplying the numerators and adding or subtracting the denominators	$\frac{3}{4}\pm\frac{2}{5}=\frac{6}{9}\left[\frac{6}{-1}\right]$
E6	Failure to determine the equivalent fractions	$4\frac{3}{10}\pm 6\frac{1}{2}+5\frac{3}{4}$ *to* $4\frac{3}{20}\pm 6\frac{1}{20}+5\frac{3}{20}$
E7	Failure to change improper fraction to mixed fraction	$\frac{43}{7}$ *to* $6\frac{3}{7}$
E8	Failure to change a mixed fraction to an improper fraction	$4\frac{4}{10}$ *to* $6\frac{40}{10}$
E9	Computation errors - Failure to add, subtract or multiply properly	
E10	Failure to write a mixed number correctly	$4\frac{4}{7}+6\frac{8}{5}$
Blank	Question not attempted at all	-
Incomplete answer	Attempted but not reached the correct answer	-

E, error.

TABLE 2: Errors on equivalent fractions.

Question	Equivalent fraction								Total
	E1	%	E2	%	E3	%	E4	%	
(a) 2/3 = x/18	3	6	24	52	9	20	9	20	45
(b) 4/5 = 16/x	3	3	32	68	7	15	4	9	46

E, errors.

In question (g), learners, like in question (e), used two approaches to solve the problem. Some added the whole numbers independently and then the fractions independently, while others changed or attempted to change the mixed numbers to improper fractions before adding. Error E5.1 was predominant, with 59% of the learners making this error. This category was followed by error E6, with 20% of the learners making this error. Table 4 also shows other errors that were made in question (g). Seven per cent of the learners made E8 errors; E9 errors were made by 5% of the learners, while 3% of the learners made E5.2 errors. Four per cent of the learners were not able to complete their solutions.

Subtraction of fractions: Questions (d), (f) and (h) were problems with the subtraction of fractions. In question (d), learners were required to subtract common fractions involving two terms with unrelated denominators. Question (f) asked them to subtract mixed numbers involving two terms with unrelated denominators. In (h), they had to subtract mixed numbers involving three terms with unrelated denominators. The test items with unrelated denominators were used to check if the learners could apply the concept of equivalent fractions in the addition and subtraction processes. The results of the errors made by learners in the subtraction of fractions are shown in Table 4.

In question (d), most learners made error E5.1. Seventy-three per cent of the learners treated the fraction components as independent whole numbers, where they would carry out subtraction on numerators and then on denominators. For example, one learner gave this answer: $\frac{1}{4}-\frac{1}{3}=\frac{0}{1}$.

TABLE 3: Addition of fractions – Errors made by the learners.

Question	Addition of fractions																		Total
	E5.1	%	E5.2	%	E5.3	%	E5.4	%	E6	%	E7	%	E8	%	E9	%	IN	%	
(c) 3/4 + 2/5	53	66	6	7	0	-	8	10	11	13	0	-	0	-	1	1	3	4	82
(e) 2 3/4 + 2 1/2	52	60	7	8	0	-	2	2	12	14	0	-	7	8	5	6	4	3	87
(g) 4 4/10 + 6 1/2 + 5 3/4	58	59	3	3	0	-	2	2	20	20	0	-	7	7	5	5	4	4	99

E, errors; IN, incomplete answer.

TABLE 4: Errors made in the subtraction problems.

Question	Subtraction of fractions																		Total
	E5.1	%	E5.2	%	E5.3	%	E5.4	%	E6	%	E7	%	E8	%	E9	%	IN	%	
(d) 1/4 -1/3	64	73	5	6	0	-	5	6	10	11	0	-	0	-	2	2	2	2	88
(f) 1 3/4-11/3	51	57	10	11	0	-	4	4	11	12	0	-	6	7	3	3	4	5	89
(h)7 2/3 - 3 7/S - 11/2	51	54	3	3	0	-	2	2	21	22	1	1	7	7	5	5	5	5	95

E, errors; IN, incomplete answer.

Out of 10 learners, 11% of the total participants, made error E6. These learners faced challenges with the equivalence of fractions when they had determined the lowest common multiple of the denominators of the fractions. In categories E5.2 and E5.4, 6% of the participants made errors. Two per cent of the learners made error E9 and 2% did not complete the solutions.

In question (f), 57% of the learners made errors E5.1, 12% made E6 errors, 11% made E5.2 errors, 7% made E8 errors and 3% made E9 errors. Five per cent of the learners did not complete the solution. In question (h), 51 learners (54%) made E5.1 errors; 22% of the learners had challenges with equivalence in fractions (category E6). E8 errors were made by 7% of the learners, while 5% made E9 errors. Five learners tendered incomplete solutions.

Looking at the three subtraction problems, it is noticeable that E5.1 was the learners' most prevalent error. In questions (f) and (h), there was a decrease in the E5.1 errors from 73% in question (f) to 54% in question (h), while there was an increase in other errors. For example, error E6 rose from 11% in question (f) to 22% in question (h). Also of particular interest was the emergence of error E8 in questions (f) and (h). Seven per cent of the learners in each question made this error. In error E8, learners struggled to convert mixed numbers to improper fractions.

Computational errors (E9) also increased from 2% in question (d) to 5% in question (h). The researchers posit that this may have been caused by a lack of confidence among the learners in working with fractions, especially when two or more terms are involved.

So far, this section has looked at the errors made by the test participants. The results are summarised in Table 5.

The most prevalent error was E5.1, with 52% errors made in the test. In this error, learners treated fraction components as separate entities and considered them independent whole numbers. Fourteen per cent of the errors were on equivalence in fractions; this was category E6. The researchers believe a lack of understanding of equivalence in fractions could have contributed to the learners' E5.1 errors.

TABLE 5: A summary of the errors found in the learners' test answers.

Error code	Total errors	%
El	6	0
E2	56	9
E3	16	3
E4	13	2
E5.1	329	52
E5.2	34	5
E5.3	0	0
E5.4	23	4
E6	35	14
E7	1	0
E8	27	4
E9	20	3
E10	0	0
IN	23	4
Total	**633**	**100**

E, errors; IN, incomplete answer.

As mentioned earlier, the researchers used the test to investigate the learners' level of understanding of addition and subtraction of fractions. The results show that learners are battling with the addition and subtraction of fractions, as evidenced by the various errors they made on the test. This is also evident in the comments some learners wrote next to their work (answers). For example, Figure 2 shows what learner 8R19 wrote to express her experience when solving the addition and subtraction of fractions involving three terms.

The learner expressed her frustration in working with fractions. For instance, in question (g), the learner after attempting the question, made these comments: 'This was seriously exhausting. I just wish to know these fractions more. This was seriously confusing and frustrating my emotions'. Her responses show she has challenges with the concept of a fraction and, hence, the independent whole number errors made.

Discussion of the results

Errors made by learners in equivalent fractions

The test results showed that learners battled with equivalent fractions. In question (a), 53% of the learners got the correct answer, while in question (b), 52% got the correct answer. As indicated in the results section, learners made errors

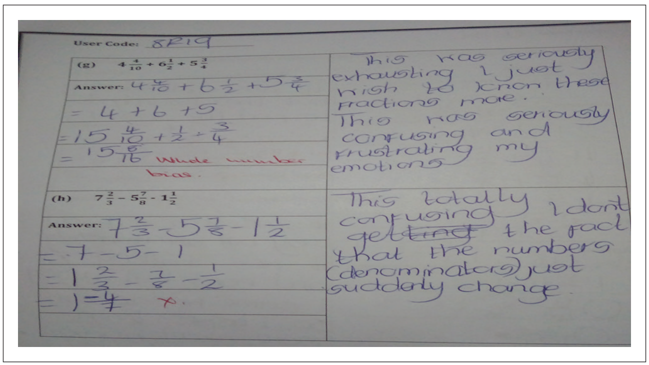


FIGURE 2: Learner 8R19's comments on her experiences in the addition and subtraction of fractions.

ranging from applying the difference between numerator and denominator in the complete fraction to the fraction with a missing value, failure to use the cross-multiplication concept in solving the equivalent fractions problems, adding the numerator and the denominator of the complete fraction to multiplying the numerator and the denominator of the complete fraction. Most learners made errors when they tried to use the cross-multiplication method to solve for a missing component in one of the given fractions in a problem. According to the SOLO taxonomy adopted by the researchers, the learners who got wrong answers on equivalence were struggling at the unistructural level; that is, failure to calculate the unknown value and recognise the restrictions associated with it when it is in the denominator position. This may point to the teaching approaches used in the teaching and learning of fractions, which emphasise procedural steps at the expense of conceptual understanding.

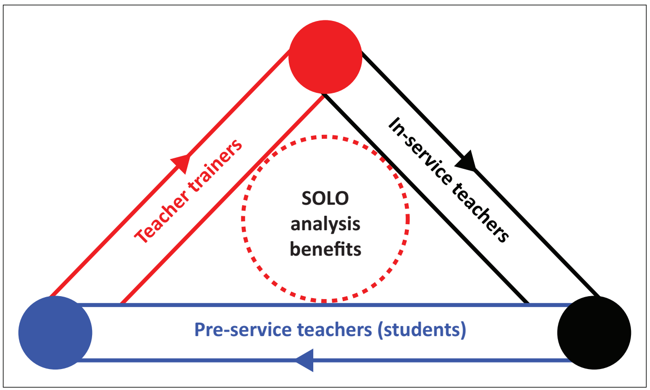
Errors made by learners in the addition and subtraction of fractions

The researchers placed the addition and subtraction of simple fractions involving two terms at the multistructural level of the SOLO taxonomy. At this level, learners were expected to be able to add and subtract simple fractions with related and unrelated denominators confidently. The results show that some learners failed to do the fractions' addition and subtraction correctly. Learners' challenges were observed in fractions with related denominators and where denominators were unrelated. The most prevalent error made by the learners was adding or subtracting the numerators and the denominators (52% in the test). The learners treated the components of the fractions as whole numbers. This error was found in the addition and subtraction of simple fractions involving two terms and mixed numbers involving two and three terms (at the relational and the extended – abstract levels). At the relational level, learners were expected to competently carry out addition and subtraction of mixed numbers with related and unrelated denominators involving two terms. In carrying out the operations, they were also expected to relate the concept of improper fractions and the lowest common multiple concept with the addition and subtraction of fractions. The errors made by the learners are confirmed by the work of Siegler et al. (2013) on the learner whole number bias challenges in addition and subtraction of fractions. This strongly indicates that learners need support in developing the concepts of fractions and mathematical operations on fractions.

The challenges in equivalent fractions also appeared in the addition and subtraction of fractions. This was indicated by error type E6 in the error analysis. Some learners (9%) failed to determine the common denominator during the addition or subtraction of fractions in the test. This shows the lack of conceptual understanding of fraction equivalence, which is critical in successfully carrying out addition and subtraction operations. As shown by the test results, learners with difficulties in equivalent fractions tended to have challenges in addition and subtraction. However, some learners got the answers on equivalent fractions correct, but all the other six questions on addition and subtraction of fractions were wrong. Jigyel & Afamasaga-Fuata'I (2007) argue that a good understanding of equivalent fractions is a foundation for a better understanding of operations with fractions. The results here show that learners did not connect the equivalent fractions concepts with the common denominator ideas in addition and subtraction of fractions. They could not extend their knowledge of equivalent fractions to addition and subtraction operations. According to the SOLO taxonomy, this demonstrates a lack of relational competency, where learners cannot relate different pieces of knowledge to the equivalent body of knowledge on addition and subtraction of fractions in this case. Makhubele (2021) concurs and posits that such learners lack an understanding of the relationship and interconnectedness of ideas to embrace the whole concept of fractions and the inherent procedures. The researchers argue that this may be attributed to the teaching and learning approaches that fail to link the concepts. Pant (2019) observed that teachers are generally focussed on imparting procedural knowledge to the detriment of conceptual knowledge. It is important to keep track of the learners' learning journey on the concepts that are being taught. 'By incorporating the SOLO taxonomy into teaching, educators can gradually raise students' level of thinking from unistructural to the abstract level, thereby encouraging deeper, more conceptual understandings' (Main 2024:p1).

Implications

The use of the SOLO taxonomy framework plays an important role in the teachers' understanding of the learners' levels of fraction understanding. This analysis maps the difficulty levels to provide a concentrated focus for the teacher and the teacher trainer to prepare the in-service teacher to provide a better graduate back into the system for practice. It also helps every teacher to understand the errors made by learners or students (Maseko et al. 2019). For more emphasis, we will quote Main (2024) again who draws this assertion: 'By incorporating the SOLO taxonomy into teaching, educators can gradually raise students' level of thinking from uni-structural to the abstract level, thereby encouraging deeper, more conceptual understandings'. The researchers also identified key people who will benefit from



SOLO, structured observed learning outcomes.

FIGURE 3: The collective approach to help learners with problems in fractions.

using the SOLO taxonomy analysis to improve the closure of the fraction knowledge gaps. These are shown in Figure 3.

A problem shared is a problem half-solved. We recommend that in-service teachers share with each other their concerns about the present learners in class within the school across the grade (horizontal collaboration). They also need to share with those grades below and grades above (1 lower and 1 up) to help support their learners better (vertical collaboration). The study showed that learners at Grade 8 had challenges that emanated from the previous grades, for example, the concept of equivalent fractions starts at the lower grades. The University and NGO teacher trainers could conduct workshops for in-service teachers and share knowledge on the SOLO application in error analysis with pre-service teachers in classes or workshop sessions.

Conclusion

Using the SOLO taxonomy, the researchers identified where each learner was in their learning journey on fractions. When all teachers apply the SOLO taxonomy in analysing their learners' levels of understanding fractions, they can design learning experiences relevant to each learner's level of understanding and assist them in advancing to higher levels of fraction knowledge and higher confidence to work and teach fractions.

Acknowledgements

Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

A.D. wrote the first draft of the manuscript and then received input from J.S.M. to compile the final draft of the article.

Funding information

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data availability

The data supporting the findings of this study are available from the corresponding author, A.D., upon reasonable request.

Disclaimer

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