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Mathematics teachers' zone of mathematical competence

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Introduction

Elementary mathematics is the cornerstone of the main strands of mathematics that students encounter at later stages of their education path. Research asserts that students who have acquired good conceptual knowledge of mathematics at elementary stages continue to perform better at later stages of their mathematical engagements. However, for learners to acquire critical mathematics knowledge from the inception stage of primary mathematics, teachers at that level must be highly grounded in the mathematics they teach. The mathematics knowledge implied here is the content knowledge (CK) which is the knowledge of mathematics they teach. At any given phase, the mathematics content is made up of numbers, operations and relationships; patterns, functions, and algebra; space and shape (geometry); measurement; and data handling (Department of Basic Education 2010).

In reference to mathematics instruction, CK has been split into common content knowledge (CCK) and specialised content knowledge (SCK). Hill et al. (2008:78) define CCK and SCK as the commonly held mathematical knowledge that anyone who knows mathematics would possess, and the mathematics knowledge specific to the teaching of the subject matter, respectively. Lee and Lust (2008) expand on SCK as 'the experiential knowledge and skills acquired through classroom experience'.

The immediate question mathematics scholars would ask is, what are the other knowledges critical to effective mathematics instruction in schools at any given phase? The knowledge of how to teach the content, the instructional knowledge, commonly known as pedagogical knowledge is both inert and taught. Pedagogical knowledge has some element of inertness because all living species are born with the ability to instruct or inform on what they know. The key word here is 'what they know' and that is where the knowledge of the content, in this instance of mathematics matters. The other component of pedagogical knowledge is that it must be taught. The need for those who know the content (mathematics, science, engineering, technology, history, medicine, language) to be taught on how to teach the content is because research (Shulman 1986) has established that knowing the content or the subject matter is not equivalent to the ability to teach it effectively. Mathematics teachers are taught how to teach mathematics in institutions of higher learning, and this established itself into the pedagogical CK, the knowledge of how to teach the mathematics content (Numbers, Operations, and Relationships; Patterns, Functions, and Algebra; Space and Shape [Geometry]; Measurement; and Data Handling).

The zone of mathematical competence

While the two knowledges, CK and pedagogical content knowledge (PCK), are critical for effective instruction, teachers need other knowledges for them to be wholesomely effective in and outside mathematics classrooms. The content is made up of various concepts from which teachers develop aims and objectives of every lesson. For teachers to be wholesomely effective, they need the knowledge of the concepts embedded in each mathematical strand known as conceptual knowledge. Schneider and Stern (2010:178) describe conceptual knowledge as 'providing an abstract understanding of the principles and relations between pieces of knowledge in certain domains'. This knowledge is pivotal to the teacher's ability to define the concepts and outline how they are applied in the context. The other knowledge is how to procedurally execute and arrive at a correct mathematical response to a problem, known as procedural knowledge (PK). I have described it in my article (Luneta 2014) as the learners' ability to respond to questions by using specific procedures or operations, particular formulas and methods, however, without providing explanations to their answers as to why certain formula, method or operations were used. Procedural fluency (McCormick 1997; Star 2002), the effectively 'knowing how to do it' is essential when teachers possess PK. Once instructors have acquired the knowledges of mathematics and how to teach it effectively, including

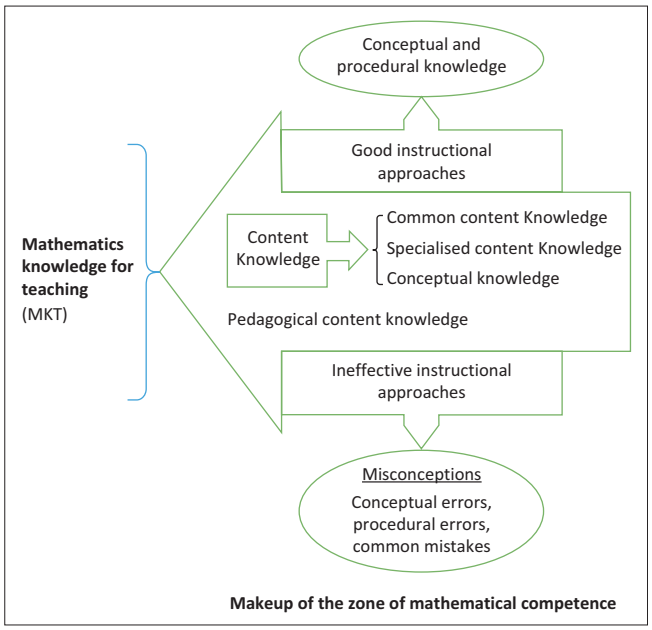

FIGURE 1: The zone of mathematical competence.

PCK and SCK, Hill et al. (2008) assert that they then have attained mathematics knowledge for teaching (MKT). I term the embellishment of Mathematics teachers that are informed by the effective and appropriate knowledge bases of mathematics and how to teach it in various context as being in the zone of mathematical competence (ZMC). In essence, effective Mathematics teachers are those that persistently flourish in the ZMC. This is a sphere that good teachers of mathematics ought to be operating in if they are to be persistently effective.

Misconceptions and the associated errors

There exists a relationship between the ZMC and misconceptions, and their associated errors. Michael (2001:11) defines misconceptions as 'conceptual or *reasoning difficulties* that hinder students' mastery of any discipline' while Drews (2005:18) identifies misconceptions as 'a misapplication of a rule, an over- or under-generalization, or an alternative conception of the situation'. Luneta (2008:386) defines errors as 'simple symptoms of the difficulties a student is encountering during a learning experience'. Errors can therefore, according to Swan (2001:150), be the result of 'carelessness or misinterpretation of symbols or text'. Misconceptions which essentially are a result of misunderstanding of the concept being taught are displayed as errors in written or spoken form. Misconceptions are best established through error analysis or interview with the learners. It is worth noting that errors are a result of specific misconceptions, hence misconceptions and associated errors. There are three types of errors that are on account of misconceptions. Conceptual errors are because of the lack of understanding of the concepts being taught or explained and procedural errors are because of the lack of knowledge of the procedure to arrive at the correct answer. Other scholars have related procedural errors to application errors, inability to apply the provided or correct formula in responding to a problem. Careless errors are easily identified and often corrected by the learner.

Conclusion

The ZMC (Figure 1) is made up of the teacher's knowledges, CK, PK, PCK, CCK, SCK, conceptual knowledge and ultimately MKT. Mathematics teachers who possess these knowledges often develop and execute effective instructional approaches that enable learners to acquire conceptual and procedural knowledges. The teachers who lack these knowledges often teach in ways that are ineffective, culminating in learners' lack of understanding what is being taught, which results in misconceptions and the associated errors. Studies show that there are few teachers who flourish in the ZMC and teach mathematics effectively.

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